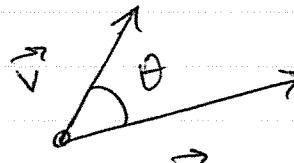


Review of Dot product (Sec. 12.3)

(2-1)

① Definition / formula



$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$\langle u_1, u_2, u_3 \rangle$ $\langle v_1, v_2, v_3 \rangle$

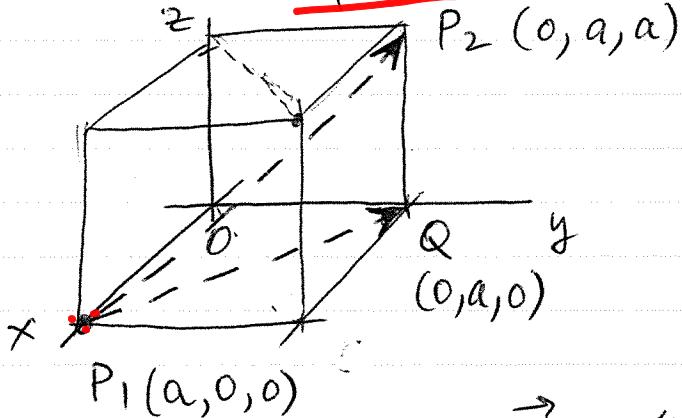
$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos \theta \quad (\star)$$

Note 1: the dot product is a scalar, not a vector.

Note 2: $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

Formula (A) can be used to find the angle between vectors.

Ex. 1 Find the angle between a diagonal of a cube and its projection onto one of the faces.



Sol'n:

1) State formula:

$$\cos \theta = \frac{\vec{P}_1 \vec{P}_2 \cdot \vec{P}_1 \vec{Q}}{|\vec{P}_1 \vec{P}_2| \cdot |\vec{P}_1 \vec{Q}|}$$

2) $\vec{P}_1 \vec{P}_2 = \langle 0-a, a-0, a-0 \rangle$

$$= \langle -a, a, a \rangle$$

$$\vec{P}_1 \vec{Q} = \langle 0-a, a-0, 0-0 \rangle = \langle -a, a, 0 \rangle$$

$$3) |\vec{P_1 P_2}| = \sqrt{(-a)^2 + a^2 + a^2} = \sqrt{3}a$$

2-2

$$|\vec{P_1 Q}| = \sqrt{(-a)^2 + a^2 + 0^2} = \sqrt{2}a$$

$$\vec{P_1 P_2} \cdot \vec{P_1 Q} = -a \cdot (-a) + a \cdot a + a \cdot 0 = 2a^2.$$

$$\text{So } \cos \theta = \frac{2a^2}{\sqrt{3}a \cdot \sqrt{2}a} = \frac{2}{\sqrt{3} \cdot \sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}}.$$

$$\theta = \arccos \frac{\sqrt{2}}{3} \approx 35^\circ.$$

(See also Ex. 3 in book).

② Dot product & length

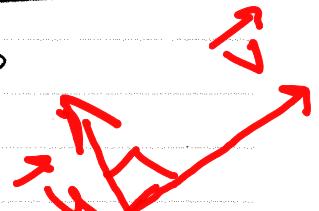
$$\boxed{\vec{u} \cdot \vec{u} = u_1 \cdot u_1 + u_2 \cdot u_2 + u_3 \cdot u_3 = u_1^2 + u_2^2 + u_3^2 \\ = |\vec{u}|^2}$$

For unit coord. vectors: $|\vec{i}|^2 = |\vec{j}|^2 = |\vec{k}|^2 = 1$
 $\vec{i} \cdot \vec{i}$ etc.

③ Dot product & orthogonal vectors

$$(A) \rightarrow \vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos \theta \Rightarrow$$

$$\boxed{(\vec{u} \perp \vec{v}) \Leftrightarrow \vec{u} \cdot \vec{v} = 0}$$



This is a practical TEST if $\vec{u} \perp \vec{v}$.

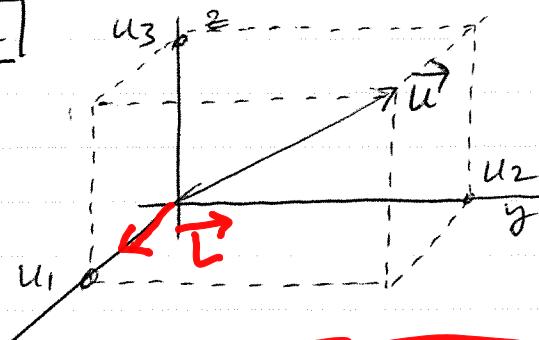
In particular:

$$\boxed{\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0}.$$

④ Dot product & projections

2-3

a



Let's compute

$$\vec{u} \cdot \vec{i} =$$

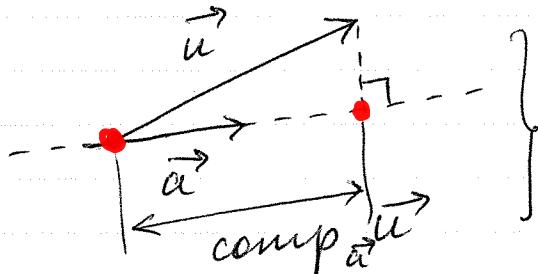
$$\langle u_1, u_2, u_3 \rangle \cdot \langle 1, 0, 0 \rangle$$

$$= u_1 \cdot 1 + u_2 \cdot 0 + u_3 \cdot 0 = u_1.$$

So:

$$\left. \begin{aligned} \vec{u} \cdot \vec{i} &= u_1 \\ \vec{u} \cdot \vec{j} &= u_2 \\ \vec{u} \cdot \vec{k} &= u_3 \end{aligned} \right\}$$

scalar projections of
 \vec{u} on coord. axes.
also called
"components"



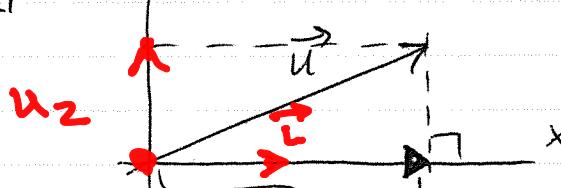
More generally,
component of \vec{u}
on any vector \vec{a} :

$$\text{comp}_{\vec{a}} \vec{u} = \vec{u} \cdot \vec{a}^*$$

unit vector
along \vec{a}

$$\vec{a}^* = \frac{\vec{a}}{|\vec{a}|}$$

b



$$\vec{u}_1 \cdot \vec{i} = \text{proj}_i \vec{u} - \text{vector projection}$$

of \vec{u} on \vec{i}
(or on x-axis).

Similarly, $\vec{u}_2 \cdot \vec{j} = \text{proj}_j \vec{u}$.

$$\text{proj}_{\vec{a}} \vec{u} = \vec{u}_1 \cdot \vec{a} = (\vec{u} \circ \vec{a}) \cdot \vec{a}.$$

(2-4)

vector projection of \vec{u} on \vec{a}

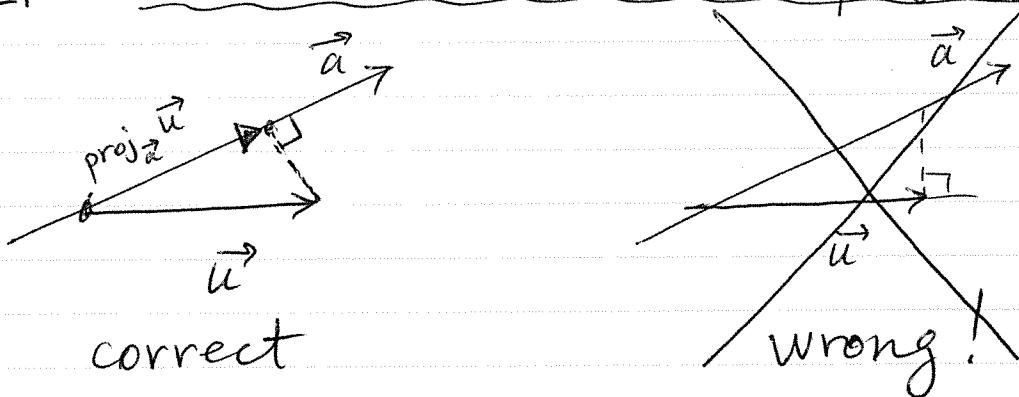
$$\text{proj}_{\vec{a}} \vec{u} = (\vec{u} \circ \vec{a}^*) \cdot \vec{a}^* = \frac{\vec{u} \circ \vec{a}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{(\vec{u} \circ \vec{a}) \cdot \vec{a}}{|\vec{a}|^2}.$$

arbitrary vector

Conclusion: the main use of dot product
is to compute projections!

c) Common mistake about projections:



Projection is always SHORTER than the original vector!

HW 3: TF @ end of Ch. 12 (p. 842): 1(3) 19, 8

See. 12.3 : 1, 14 - def.;
7, 9 - basic properties;
20, 55 - angle between vectors
23 - || or \perp
④ ⑩, ⑪, ⑯ - projections.

Review Ex. p. 842 #9 - angle/diags in cube

EC #1 Sec. 12.3 #58.