

Calculus III

(1-1)

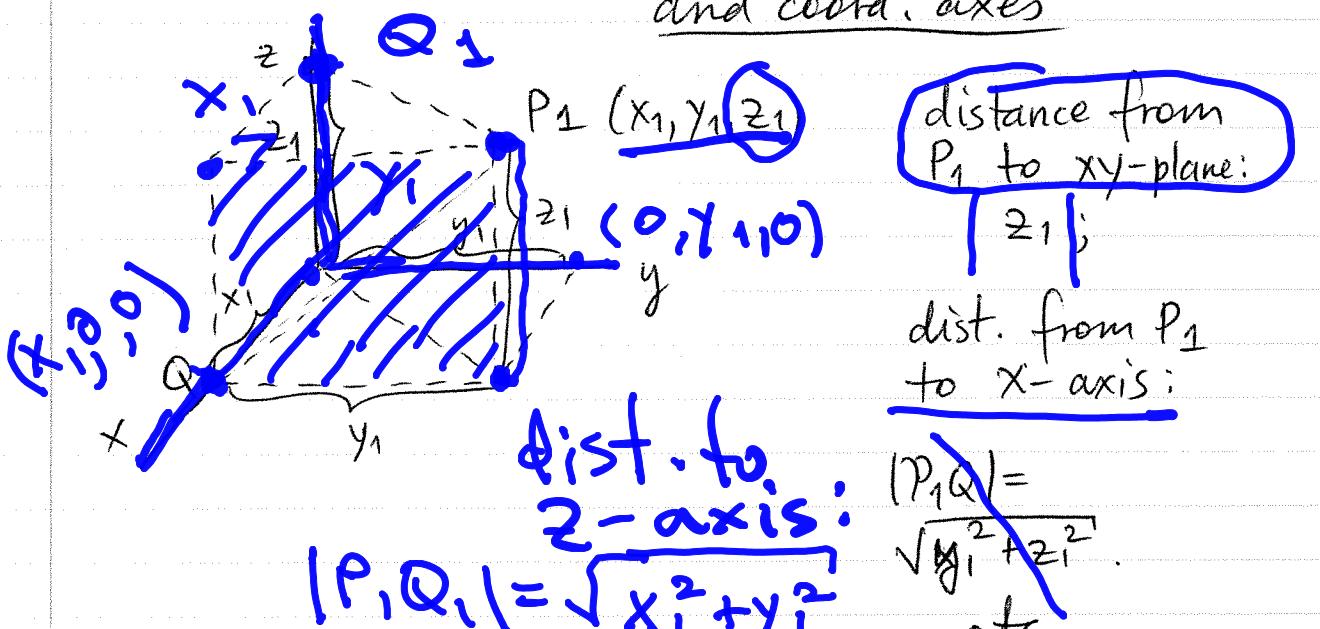
J. Stewart, 8th Ed. (2017), Early Transcendentals

Lecture 1: Review of coordinates & vectors

① Coordinates

(Sec. 12.1)

- a Definition, distance to coord. planes and coord. axes

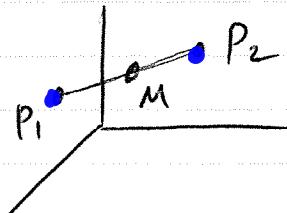


- b Planes \parallel to coord. planes

$x = x_1 \leftarrow$ plane \parallel zy -plane,
 x_1 units away from it
 (contains Q , P_1 in the figure).

- c Midpoint of segment

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$$



d

Distance between points :

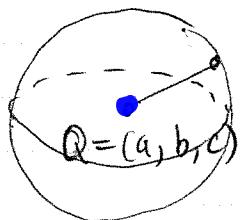
1-2

$$|P_1 P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(like the Pythagorean Thm. in 3D).

e

Sphere : all points equidistant from center $Q(a, b, c)$:



$P(x, y, z)$

$Q(a, b, c)$

$$|PQ| = R = \text{const}$$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = R$$

|PQ|

2

Vectors (Sec. 12.2)

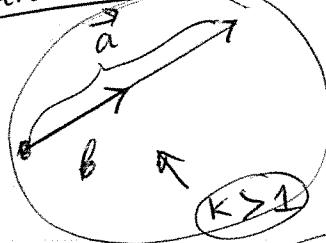
a

Parallel vectors

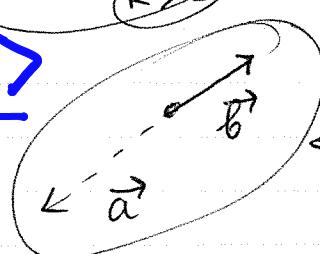
$$(\vec{a} \parallel \vec{b}) \Leftrightarrow (\vec{a} = k \cdot \vec{b})$$

for some scalar k .

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$



$$k = \frac{\text{length } \vec{a}}{\text{length } \vec{b}} = \frac{|\vec{a}|}{|\vec{b}|}.$$



$$k < 0. \quad \langle 1, 2, 3 \rangle \\ \langle 2, 4, -6 \rangle$$

b

Sum & difference of vectors

Algebraically:

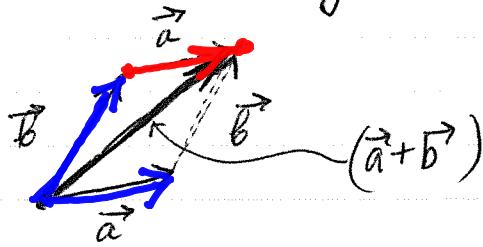
same DPP.

$$\vec{a} \pm \vec{b} = \langle a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \rangle.$$

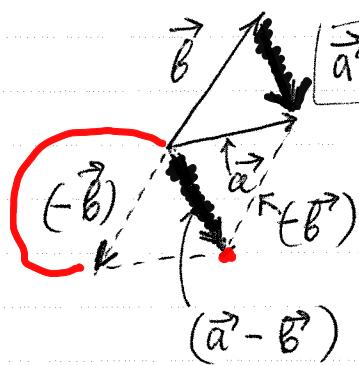
$$\langle 2, 4, -6 \rangle \neq 2 \cdot \langle 1, 2, 3 \rangle \\ \neq -2 \cdot \langle 1, 2, 3 \rangle$$

1-3

Geometrically:



$$\textcircled{a} \quad \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$



Mnemonic rule:

[points from end of \vec{b} to end of \vec{a}]

3 vectors in same plane

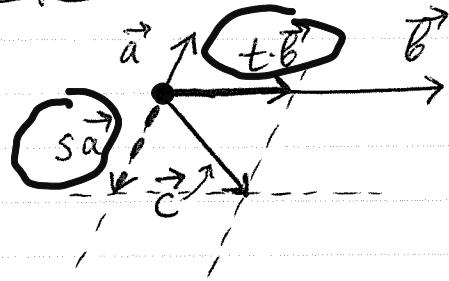
Suppose $\vec{a}, \vec{b}, \vec{c}$ lie in same plane,
and $\vec{a} \parallel \vec{b}$. Then:

$$\vec{c} = s \cdot \vec{a} + t \cdot \vec{b}$$

for some scalars s, t .

Geometric proof:

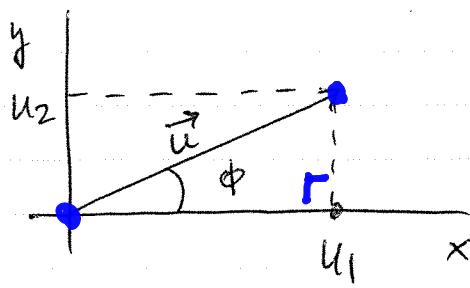
This will work
for any \vec{c} .



d

length & angle in 2D

1-4



$|\vec{u}| = \text{length of } \vec{u}$

$$\vec{u} = \langle u_1, u_2 \rangle$$

$$= \langle |\vec{u}| \cos \phi, |\vec{u}| \sin \phi \rangle.$$

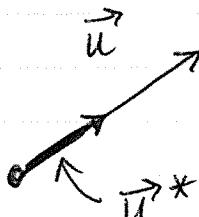
Note: Must consistently use vector notations if you mean vectors:

① $\hat{\vec{u}}$

\vec{u} ← vector; u ← scalar.

e

Unit vectors



$$\vec{u}^* = \frac{\vec{u}}{|\vec{u}|}$$

↑
unit
vector
along \vec{u} .

length
of \vec{u}

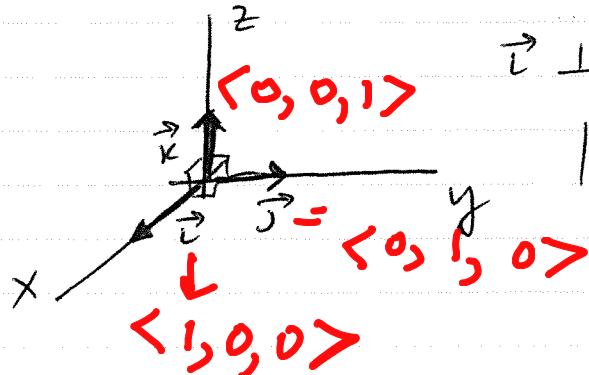
For example, a vector of length 3 and along \vec{u} is:

$$3 \cdot \vec{u}^* = \frac{3}{|\vec{u}|} \cdot \vec{u}.$$

↑ length ↑ direction

f

Unit coordinate vectors



$$\hat{i} \perp \hat{j} \perp \hat{k} \perp \hat{i}.$$

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1.$$

HW 1 (Sec. 12.1)

1-5

3, 12, 15, 23, 25, 29.
dist. ↑ sphere eq. of plane //
meaning of coords & coord. planes
distance

HW 2 (Sec. 12.2)

3, 9, 13, 43, 15 - basics, mem

25, 26 - unit vect; length/direction
31 - length & angle
45 - $\vec{c} = s\vec{a} + t\vec{b}$
47 - $|r - r_0| = 1$.