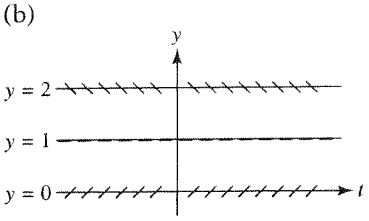
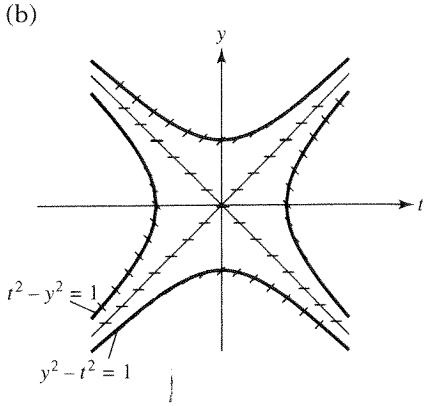
7. (a) The requested isoclines are the lines $y = 2, y = 1$, and $y = 0$.
(b)



9. (a) The requested isoclines are the hyperbolas $y^2 - t^2 = -1, y^2 - t^2 = 0$, and $y^2 - t^2 = 1$.
(b)



11. One possibility is $y' = -(y-1)^2$.
13. One possibility is $y' = \sin(2\pi y)$.
15. Direction Field F
17. Direction Field B
19. Direction Field E

CHAPTER 2

Section 2.1, page 17

1. Linear and nonhomogeneous
3. Nonlinear
5. Nonlinear
7. Nonlinear
9. Linear and nonhomogeneous
11. (a) $-\infty < t < \infty$ (b) $-\infty < t < \infty$ (c) $-\infty < t < \infty$
13. (a) $3 < t < \infty$ (b) $-2 < t < 2$ (c) $-2 < t < 2$ (d) $-\infty < t < -2$ (e) $-2 < t < 2$
15. $p(t) = -2t$ and $y_0 = 3$
17. $y(t) = 0, a < t < b$

Section 2.2, page 26

1. (a) $y = Ce^{-3t}$ (b) $y = -3e^{-3t}$
3. (a) $y = Ce^{t^2}$ (b) $y = 3e^{-1}e^{t^2} = 3e^{(t^2-1)}$
5. (a) $y = -2 + Ce^{3t}$ (b) $y = -2 + 3e^{3t}$
7. (a) $y = \frac{1}{5}e^t + Ce^{-3t/2}$ (b) $y = (e^t - e^{-3t/2})/5$
9. (a) $y = -3 + Ce^{-0.5\sin t}$ (b) $y = -3 - e^{-0.5\sin t}$
11. $y = Ct^{-4}$
13. $y = Ce^{\sin 2t}$
15. $y = Ce^{t^3+3t}$
17. $y = 0.5 + Ce^{-2t}$
19. $y = te^{-2t} + Ce^{-2t}$
21. $y = \frac{1}{4}t^2 + Ct^{-2}$
23. $y = t - 1 + Ce^{-t}$
25. (a) 2 (b) 3 (c) 1
27. $\alpha = 2$ and $y_0 = \frac{1}{4}$
29. (a) $B' = -kB, \quad B(0) = -A^*$ (b) $A(c) = A^*(1 - e^{-kc})$. $A(c)$ never exceeds A^*. (c) $c = (1/k)\ln 20$
31. $p(t) = 2, \quad g(t) = 2t + 3$
33. $p(t) = t^{-1}, \quad g(t) = t^{-1}$
35. $g(t) = 2e^t + \sin t + \cos t, \quad y_0 = -1$
37. $\lim_{t\to\infty} y(t) = -1$
39. A finite limit exists whenever $\lambda > 0$. In this case, the limit is equal to $1/\lambda$.
41. $$y = \begin{cases} 1 + 2e^{-1+\cos t}, & 0 \le t \le \pi \\ -1 + 2e^{1+\cos t} + 2e^{-1+\cos t}, & \pi < t \le 2\pi \end{cases}$$

43. $y = \begin{cases} 3e^{-t^2+t}, & 0 \le t \le 1 \\ 3, & 1 < t \le 3 \\ t, & 3 < t \le 4 \end{cases}$

45. $y = e^{t^2}\left[2 + \dfrac{\sqrt{\pi}}{2}\operatorname{erf}(t)\right]$

Section 2.3, page 37

1. (a) $Q(10) = 20(1 - e^{-0.3}) \approx 5.18$ lb

   (b) $\lim_{t\to\infty} Q(t) = 20$ and the limiting concentration is 0.2 lb/gal.

3. The required inflow rate is $r = (14{,}000/3)\ln(100) \approx 21{,}491$ m^3/min. The fraction vented per minute is $r/v = (1/30)\ln(100) \approx 15.4\%$.

5. (a) $Q(t) = 500t^2e^{-t/50}$ mg

   (b) The maximum value occurs at $t = 100$ min. The maximum concentration is about 135.3 mg/gal.

   (c) Yes, a graph of concentration versus time shows that $c(t) > 100$ for $60 \le t \le 160$.

7. (a) $t = 600$ min

   (b) $c(300) = Q(300)/V(300) = 197.5/400 \approx 0.494$ lb/gal

   (c) $0.5 - (40/700)(1/49) \approx 0.4988$ lb/gal

9. (a) $Q(0) = 0$ (b) $c_i(t) = 0.05$ lb/gal

11. (a) $Q' = (15/500)(\alpha - 1)Q$ (b) $\alpha = 1 - (1/5.4)\ln 100 \approx 0.1472$

13. (a) $Q_A' = -1000(Q_A/500{,}000)$, $Q_A(0) = 1000$
    $Q_B' = 1000(Q_A/500{,}000) - 1000(Q_B/200{,}000)$, $Q_B(0) = 0$

    (b) $Q_A(t) = 1000e^{-t/500}$ lb, $Q_B(t) = (2000/3)(e^{-t/500} - e^{-t/200})$ lb

    (c) The maximum value is attained at $t = (1000/3)\ln 2.5 \approx 305.4$ hr.

    (d) About 4056 hours, or approximately 169 days, is required.

15. (a) No, we do not expect the concentration to stabilize, since the inflow rate is varying.

    (b) $Q' = 0.6(1 + \sin t) - (3/200)Q$, $Q(0) = 10$

    (c) $Q(t) = 40 - 30e^{-(3/200)t} + (1/1.000225)[0.6(e^{-(3/200)t} - \cos t) + 0.009\sin t]$ lb

17. An oven temperature of $70 - 80/(\sqrt{15/23} - 1) \approx 485°$F

19. (a) $\theta(0) = 340°$F (b) $\theta(t) \to S_0 = 70°$F as $t \to \infty$

21. (a) $\theta(0) = 40°$F (b) $\theta(t) \to S_0 = 80°$F as $t \to \infty$

23. The times are the same.

Section 2.4, page 45

1. $P(30) = 10{,}000{,}000e^{6\ln(1.1)} = 17{,}715{,}610$

3. $t = (2\ln 3)/\ln 1.3 \approx 8.375$ weeks

5. It will take an additional 9.6 days.

7. $Q(0) = 20\sqrt{32} = 113.137\ldots$ g

9. After 45 days

11. (a) For Strategy I, $M_I = kP_0$. For Strategy II, $M_{II} = (e^k - 1)P_0$.

    (b) For Strategy I, the profit will be $500{,}000(0.3172)(0.75) = \$118{,}950$. For Strategy II, the profit will be $500{,}000(e^{0.3172} - 1)(0.6) \approx \$111{,}983$.

13. (a) $t = (5730/\ln 2)\ln(10/3) \approx 9953$ years (b) $9901 \le t \le 10{,}005$ years
    (c) $Q(60{,}000)/Q(0) \approx 7.04 \times 10^{-4}$

15. Approximately 38.9 micrograms