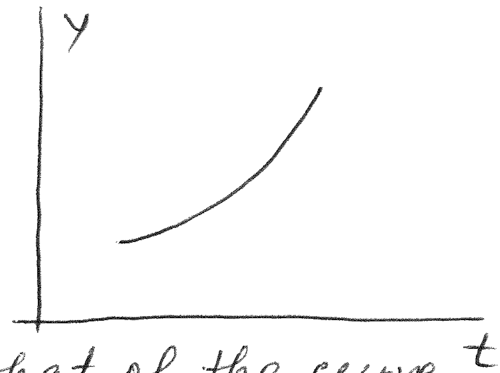
Ex. 4 worked out step by step

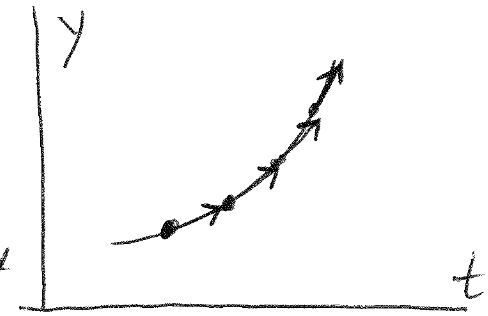
Sketch the direction field of $y' = \sin y$.

0. Motivation

Let us pretend that we know some solution $y(t)$; the curve on the right is its sketch.



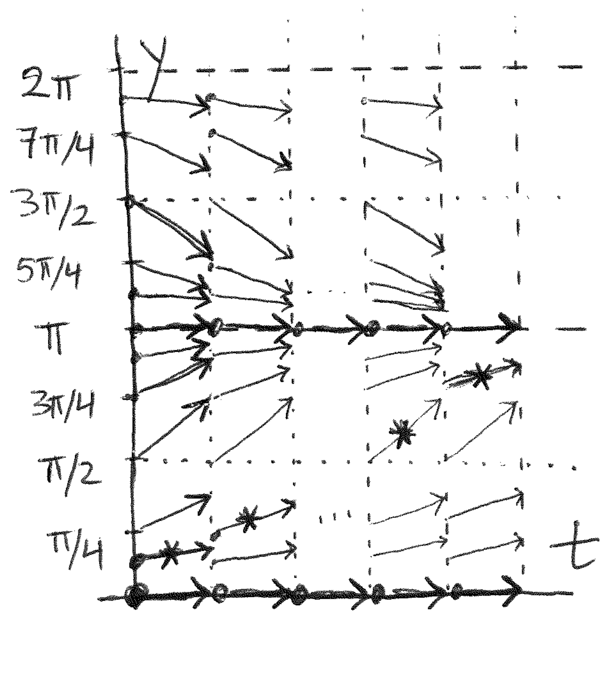
Along its length, sketch vectors whose slopes follow that of the curve. Keep the lengths of all your vectors the same. (We can call this fixed length the "unit length".)



You may recall unit tangent vectors $\vec{T}$ from Calc. III.
You can see that these vectors trace out this solution's curve (approximately).

1.

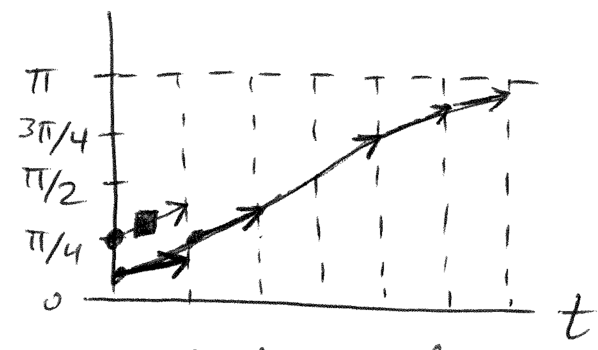
For our ODE, at a given t-value and for several representative points (t, y), sketch the "unit vector" whose slope equals y'.
(The values are listed on p. 1-8 of rec. 2.)



2. Sketch such vectors at several consecutive "t-slices".

3. Connect these vectors in some smooth way (this is rather subjective).

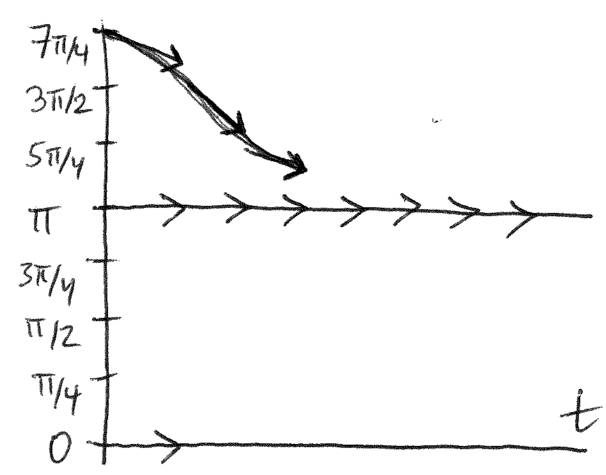
Here is an example, connecting vectors with "*" in the previous picture:



This "trajectory" is an approximate graphical representation of <u>one</u> of the solutions of the ODE.

4. Do the same for several (t_0, y) "starting" vectors, e.g., the vector with "■" above, or the vector starting at $(t_0, y = 7\pi/4)$ shown below.

This will give you the idea of what <u>various</u> solutions of the ODE will do as time increases.



$\boxed{\text{In this lies the usefulness of the direction field.}}$