Background from Algebra, Precalculus, and Calculus I

which you are required to get down cold

Since this course is about solving differential equations, you must know the solution of the *simplest* differential equation:

$$y'(t) = f(t), \qquad \text{where } f(t) \text{ is some known function.}$$

Thus:

$$\boxed{y'(t) = f(t) \qquad \Leftrightarrow \qquad y(t) = \int f(t)\,dt\,.}$$

Algebra

Quadratic equations:

(a) Roots of $ax^2 + bx + c = 0$ are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \equiv \frac{-(b/2) \pm \sqrt{(b/2)^2 - ac}}{a}.$$

(b) Special cases of the above:

- $c = 0$: then $x_1 = 0$, $x_2 = -b/a$;
- $b = 0$: then $x_{1,2} = \pm\sqrt{-c/a}$.

(c) Graphs of $y = ax^2 + bx + c \equiv a(x - x_1)(x - x_2)$ for both $a > 0$ and $a < 0$.
If you do not remember these graphs, use Mathematica or any other software to plot them.
For example, in Mathematica the command is:

```
Plot[3*(x-2)*(x-6), {x,-1, 10}]
```

Exponential functions:

$$a^{m+n} = a^m \cdot a^n, \qquad a^{m-n} = a^m/a^n;$$

$$a^{mn} = (a^m)^n = (a^n)^m\,.$$

Precalculus

Logarithms:

$$\ln(ab) = \ln a + \ln b, \qquad \ln a^n = n\,\ln a, \qquad \ln(a/b) = \ln a - \ln b, \qquad \ln\left(\frac{1}{a}\right) = -\ln a\,.$$

Exponentials and Logarithms:

$$e^{\ln a} = a, \qquad e^{\ln a + \ln b} = ab, \qquad e^{f(t)+C} = e^{f(t)} \cdot e^{C}, \qquad e^{a\,\ln t} = t^{a}.$$

Graphs of exponentials, sines, and cosines:

You must know:

- What graphs of e^{at} look like for both $a > 0$ and $a < 0$.
  Do `Plot[Exp[x], {x,-0.5, 1.5}]` etc. if you do not remember.
- How graphs of e^{2t} and e^{t} differ for $t > 0$.
- How graphs of e^{-2t} and e^{-t} differ for $t > 0$.
- Where graphs of $\sin t$ and $\cos t$ have zeros and maxima and minima.
- How graph of $\sin 2t$ and $\sin(t/2)$ differ from the graph of $\sin t$.

Basic Calculus I

Differentiation rules:

- $$(f \pm g)' = f' \pm g';$$
- $$(c\,f(t))' = cf'(t) \qquad \text{if and only if} \qquad c = \text{const};$$
- $$(f\,g)' = f'\,g + f\,g' \qquad \text{(Product Rule)};$$
- $$\frac{d\,f[u(t)]}{dt} = \frac{df}{du} \cdot \frac{du}{dt} \qquad \text{(Chain Rule)}.$$

Derivatives and integrals of basic functions:

Everywhere below $a = \text{const}$; '$+C$' is implied for all indefinite integrals.

- $$(e^{at})' = a\,e^{at}; \qquad \int e^{at}dt = \frac{1}{a}e^{at}.$$
- $$(\sin(at))' = a\,\cos(at); \qquad \int \sin(at)dt = -\frac{1}{a}\cos(at).$$
- $$(\cos(at))' = -a\,\sin(at); \qquad \int \cos(at)dt = \frac{1}{a}\sin(at).$$
- $$\begin{cases} (t^a)' = a\,t^{a-1} \\ (\ln t)' = t^{-1}\,; \end{cases} \qquad \begin{cases} \int t^a dt = \frac{1}{a}t^{a+1}, \quad a \neq -1 \\ \int t^{-1}dt = \ln|t|\,. \end{cases}$$

Fundamental Theorem of Calculus:

$$\int_{t_0}^{t} F'(t_1)\,dt_1 = F(t) - F(t_0).$$

Of course, any "letter" will work in place of $F(t)$; e.g., $y(t)$.

Calculus I / II

Integration by parts:

This corollary of the Product Rule will be helpful, but I do *not* expect it from memory:

$$\int f(t)g'(t)dt = f(t)g(t) - \int f'(t)g(t)dt$$

Alternative form of the same:

$$\int f(t)\,dg(t) = f(t)g(t) - \int g(t)\,df(t)\,.$$

Partial fraction expansion (PFE):

Although you learned of four cases of PFE, in this course we will only need the simplest one, and only at the very end (when we study Laplace Transform):

$$\frac{as+b}{(s+c)(s+d)} = \frac{A_1}{s+c} + \frac{A_2}{s+d};$$

you should review your Calculus textbook on how to find the constants A_1 and A_2.