(7) Clarifications to MUST-READ Examples 2 and 5

I do not repeat the statements of these Examples here because you must still read them. I only provide clarifications to their steps that have caused confusion to some students.

Ex. 2

$x_1 = x_3 + 3x_4$, $x_2 = -2x_3 - x_4$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_3 + 3x_4 \\ -2x_3 - x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \cdot x_3 + 3 \cdot x_4 \\ -2x_3 - 1 \cdot x_4 \\ 1 \cdot x_3 + 0 \cdot x_4 \\ 0 \cdot x_3 + 1 \cdot x_4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \cdot x_3 + \begin{pmatrix} 3 \\ -1 \\ 0 \\ 1 \end{pmatrix} \cdot x_4$$

vector form of the solution to a lin. system

Ex. 5 & 6

For Ex. 5, I'll change the book's notations

- I'll put all variables on the left and all constants on the right;
- I'll denote all variables as $x_1, \ldots$, even though the book denotes them y & z;
- For constants on the right I'll assume some arbitrary values (10, 11, 12), to clearly distinguish them from the rest.

1st l.s. in Ex. 5:

Original form:

$$\begin{aligned} 2x_1 - x_2 &= 10 \\ -3x_1 + 2x_2 &= 11 \\ x_1 + 3x_2 &= 12 \end{aligned}$$

Matrix form:

$$\begin{pmatrix} 2 & -1 \\ -3 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$$

2nd l.s. in Ex. 5:

Original form:

$$\begin{aligned} -4x_1 + 2x_2 &= 10 \\ 3x_1 + x_2 &= 11 \end{aligned}$$

Matrix form:

$$\begin{pmatrix} -4 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$$

For Ex. 6:

Original form:

$$\begin{aligned} x_1 + 3x_2 - x_3 &= 2 \\ 2x_1 + 5x_2 - x_3 &= 6 \\ 2x_1 + 8x_2 - 2x_3 &= 6 \end{aligned}$$

Matrix form:

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & 5 & -1 \\ 2 & 8 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 6 \end{pmatrix}$$