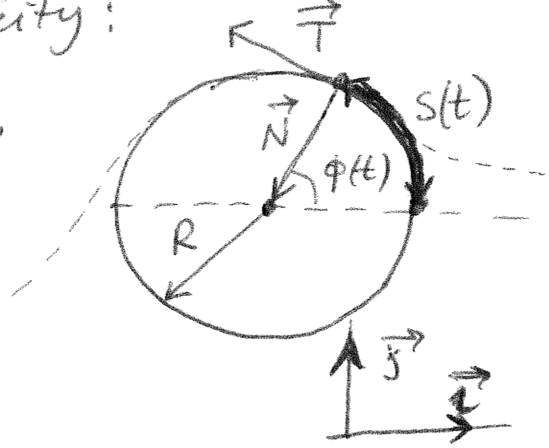
⊛ Another form of the 2nd Law of Newton for rotation: Moment of inertia & angular acceleration.

As in Example 1, assume motion in a circle (the osculating circle), but do not assume constant velocity:

$\vec{r} = R\langle \cos\phi(t), \sin\phi(t)\rangle$.

(If $\phi(t) = \omega t$, $\omega = \text{const}$, we get the case of uniform motion from Ex. 1.)



Recall from Sec. 12.4:

$$m(\vec{r}\times\vec{a}) = \underbrace{\vec{r}\times\vec{F}_{ext}}_{\vec{M}\ \leftarrow\ \text{torque}}.$$

We'll only transform the l.h.s..

Substitute $\vec{a} = \vec{T}\cdot a_T + \vec{N}a_N$:

$$\vec{r}\times(\vec{T}a_T + \vec{N}a_N) =$$

$$(\vec{r}\times\vec{T})a_T + \cancel{(\vec{r}\times\underset{\uparrow\ \parallel\vec{r}}{\vec{N}})}^{\vec{0}}a_N =$$

$$(\underset{\uparrow\ -\vec{N}\cdot R}{\vec{r}}\times\vec{T}\cdot)a_T = \underbrace{-(\vec{N}\times\vec{T})}_{\vec{k}\ \leftarrow\ \text{the vector pointing out of the page}}\cdot R a_T$$

So: $(\vec{r} \times \vec{a}) = \vec{k} \cdot R \cdot a_T$

$= \vec{k} \cdot R \cdot s''$

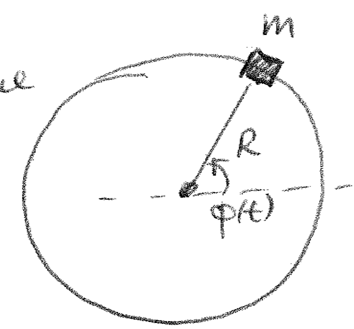
Sec. 13.3 $\;\underset{\uparrow}{=}\; \vec{k} \cdot R \cdot (\overset{\text{const}}{R}\,\varphi(t))'' = \vec{k} \cdot \underbrace{R \cdot R}_{R^2} \cdot \varphi''(t)$

Thus:

$$m\, \vec{r} \times \vec{a} = m R^2 \cdot \vec{k}\, \varphi''(t) \,, \Rightarrow$$

2nd Law of N, for rotational motion becomes:

$$\underbrace{m R^2}_{\text{moment of inertia of } \underline{\text{point}} \text{ mass } m} \cdot \underbrace{\vec{k} \frac{d^2\varphi}{dt^2}}_{\text{angular acceleration.}} = \overrightarrow{M} \leftarrow \text{torque}$$



Analogy: • Regular 2nd Law of N.:

$$m \cdot \frac{d^2 x}{dt^2} = F$$

(mass) · (acceleration) = (force)

• Rotational 2nd Law of N.:

$$\overbrace{mR^2}^{I} \cdot \frac{d^2\varphi}{dt^2} = M$$

$\begin{pmatrix}\text{moment}\\ \text{of inertia}\end{pmatrix} \cdot \begin{pmatrix}\text{angular}\\ \text{acceleration}\end{pmatrix} = (\text{torque})$.