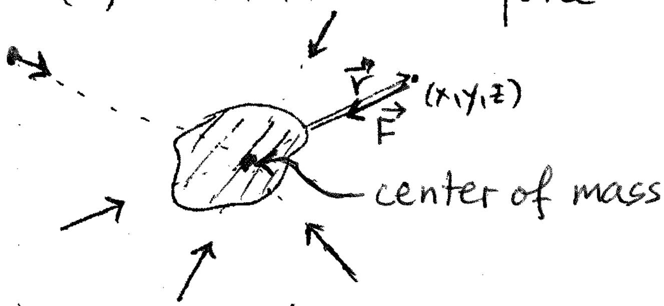
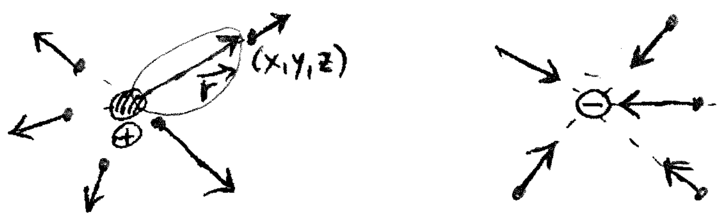
Sec. 16.1 Vector fields

Examples

(a) Gravitational force



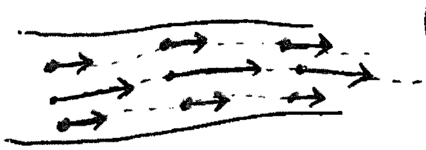
(b) Electrostatic fields



(a) & (b) are so-called central forces:

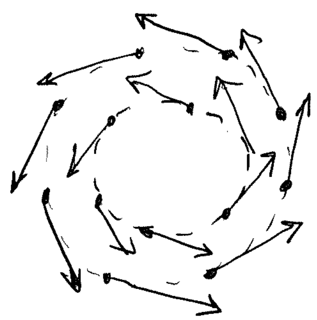
$$\vec{F}(x,y,z) = f(x,y,z)\,\vec{r} \quad \text{(vectors are directed along } \vec{r}\text{)}.$$

(c) Fluid flow



Velocity vectors at each point form a vector field. Velocity vectors are tangent to flow lines at each point.

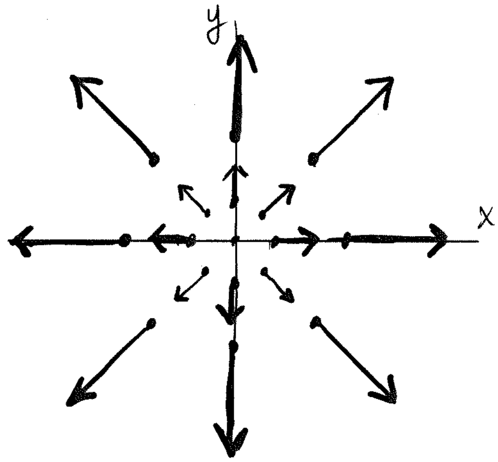
Another example of a velocity field:
Velocity of the air in a tornado.



The velocity fields in neither of these examples are central.

(d) Take any function $f(x,y,z)$ and compute $\vec{F} = \vec{\nabla} f(x,y,z)$. This gives a vector field at any point (x,y,z). Such a field (for any $f(x,y,z)$) is called a **conservative field.**
This name will be explained and used later (Sec. 16.3 and on).

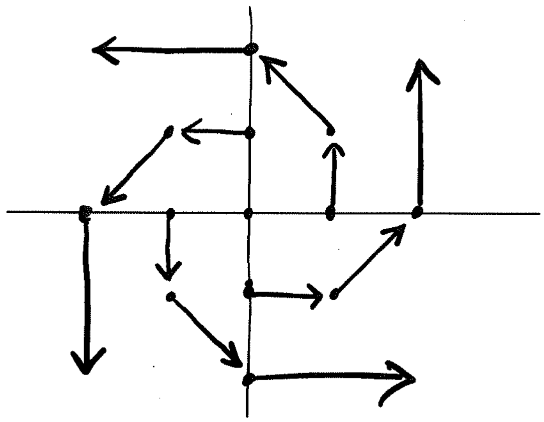
Ex. 1 Sketch the vector field $\vec{F} = x \cdot \vec{i} + y \cdot \vec{j}$.



Note: $\vec{F} = \langle x, y \rangle = \vec{r}$.
So at every point (x,y) one simply needs to draw vector $\vec{r}$. This is the vector that connects $(0,0)$ to (x,y).

An observation to be used later:
The vectors in this field diverge from a source (as the gravitational and electrostatic fields do, or the field of velocity of air in an explosion diverges from the source of an explosion).

Ex. 2 Sketch the field $\vec{F} = -y\cdot\vec{i} + x\cdot\vec{j}$.



Here we need to plot vectors of the field going one point at a time, until we notice a pattern...

@ $(1,0)$ $\vec{F} = \langle -0, 1\rangle$

@ $(0,1)$ $\vec{F} = \langle -1, 0\rangle$

@ $(-1,0)$ $\vec{F} = \langle -0, -1\rangle$; @ $(0,-1)$ $\vec{F} = \langle +1, 0\rangle$;

@ $(1,1)$ $\vec{F} = \langle -1, 1\rangle$; @ $(-1,1)$ $\vec{F} = \langle -1,-1\rangle$ etc.

An observation to be used later:

This vector field rotates (as the air in a tornado).

MUST READ Ex. 2 in book.