Ex. 6 (a) Find the form of y_p for:

$$y'' - y' = t(e^t - 2) \equiv g(t)$$

Solution: $g(t) = g_1 + g_2$, where $g_1 = te^t$, $g_2 = -2t$. So $y_p \equiv y_{p1} + y_{p2}$, where y_{p1} corresponds to g_1, y_{p2} corresponds to g_2.

1) Find y_h: $y = e^{\lambda t} \rightarrow y_h'' - y_h' = 0 \Rightarrow$

$\lambda^2 - \lambda = 0 \Rightarrow \lambda_1 = 1,\ \lambda_2 = 0.$ $\boxed{y_h = c_1 e^t + c_2}$

(since $e^{0 \cdot t} = 1$).

2) Find (y_{p1}) for $g_1 = te^t$.

According to the Table on p. 163 (see also our Ex. 2), the form of y_p is:

$$t^r \underbrace{(A_1 t + A_0)} e^t,$$

it justifies the form under $(\star)$ ....

where r is chosen so that no part of $(\star)$ is also part of y_h.

$r = 0$ does not work, since $A_0 e^t$ is part of y_h.

$r = 1$ does work, since neither $A_1 t^2 e^t$ nor $A_0 te^t$ is part of y_h.

So, the form of $y_{p1} = t(A_1 t + A_0)e^t \equiv (A_1 t^2 + A_0 t)e^t$.

A_1, A_0 can be found as in Ex. 2, 4.

3) Find (y_{p2}) corresponding to $g_2 = -2t$

According to the Table on p. 163 and our Ex. 2:

$$y_{p2} = t^r \cdot (B_1 t + B_0), \qquad (\star\star)$$

where r is chosen so that no part of $(\star\star)$ is also part of y_h.

$r=0$ does not work, since then B_0 is part of y_h.

$r=1$ does work since neither $B_1 t^2$ nor $B_0 t$ is part of y_h.

So, the form of $y_{p2} = t(B_1 t + B_0) \equiv B_1 t^2 + B_0 t$.

Finally, recall that $y_p = y_{p1} + y_{p2}$.

Ex. 6(b) (in some sense, inverse to Ex. 6(a)).

Let the DE be $y'' + \alpha y' + \beta y = g(t)$ with $\alpha, \beta = \text{const}$, $g(t) = te^t - 2t + e^{3t}$. If the form

$y_p = (a_2 t^2 + a_1 t)e^t + (b_2 t^2 + b_1 t) + ce^{3t}$,

find α, β.

Sol'n: 1) The term $g_1 = te^t$ gave rise to $y_{p1} = (a_2 t^2 + a_1 t)e^t$, $\Rightarrow$ neither $t^2 e^t$ nor te^t is part of y_h. However, $1 \cdot e^t$ must be part of y_h, since otherwise it would have been included into y_{p1}. So $\lambda_1 = 1$ (for $y_h = e^{1 \cdot t}$).

2) Term $g_2 = -2t$ gave rise to $(b_2 t^2 + b_1 t)$, $\Rightarrow$ neither t^2 nor t is part of y_h. However, 1 must be a part of y_h, since otherwise it would have been included into y_{p2}.

So, $\lambda_2 = 0$ (for $y_h = e^{0 \cdot t} = 1$).

3) $g_3 = e^{3t}$ gave rise to ce^{3t}, $\Rightarrow$ according to the Table on p. 163, e^{3t} is <u>not</u> part of y_h (i.e. $\lambda \neq 3$). Actually, we already know this, since a 2nd-order DE can only have 2 roots, and we have already found both of them: $\lambda_1 = 1$, $\lambda_2 = 0$.

4) Finally, find α, β.

$(\lambda - \lambda_1)(\lambda - \lambda_2) = (\lambda - 1)(\lambda - 0) = \lambda^2 - \lambda$ $\Rightarrow$ the l.h.s. of the DE is $y'' \underbrace{- y'}_{\alpha y'} + \underbrace{0 \cdot y}_{\beta y}$, $\Rightarrow$

$\alpha = -1$, $\beta = 0$.