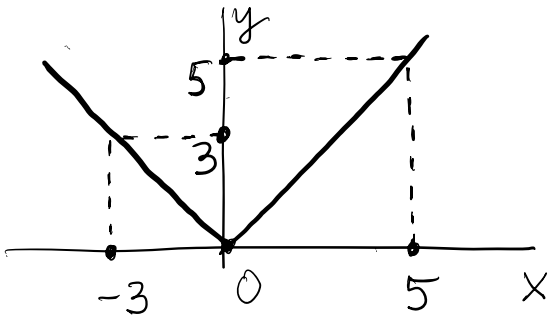
Working with the absolute value, $|\ldots|$.

11-A1

(1) Function $|x|$



Shown is the graph of $y = |x|$.
The notation $|x|$ means this:

- When $x \geq 0$, $|x| = x$.
  For example, $|5| = 5$.
- When $x < 0$, $|x| = -x$.
  For example, $|-3| = 3 = -(-3)$.

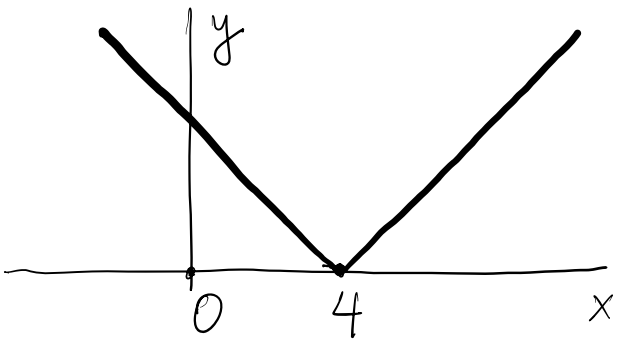
Importantly, note that the "$-$" in $|x| = -x$ when $x < 0$ does not mean that "$-x$" is negative! On the contrary, it means that the negative number x is multiplied by -1 to make the product, $-1 \cdot x \equiv -x$, positive. See the example about $|-3|$ above: $-(-3) = 3$, a positive number.

Ex. 1 Suppose we know that $|y| = x^2 + 1$ and that $y(2) = -5$. Find $y(x)$.

Sol'n: We know that:
$|y| = y$ when $y \geq 0$; and
$|y| = -y$ when $y < 0$.
In our case, $y(2) = -5 < 0$; therefore, we must choose $|y| = -y$. (Note that $-(-5) = 5 = |-5|$.)
Then: $-y = x^2 + 1 \Rightarrow y = -(x^2 + 1)$.

11-A2

② Function $|x-a|$, a = some const.



Shown is the graph of $y = |x-4|$.
We may call $(x-4)$ by another letter, say z: $(x-4) \equiv z$. Then $y = |x-4| \equiv |z|$, and we can appey everything from the previous topic to this simpler function. So:

- $z > 0 \Rightarrow |z| = z$. In terms of x, this becomes: $x-4>0 \Rightarrow |x-4| = x-4$
- $z < 0 \Rightarrow |z| = -z$. In terms of x, this becomes: $x-4<0 \Rightarrow |x-4| = -(x-4)$.

So, if $y = |x-4|$, then $y(1) = |1-4| = |-3| = 3$;
$y(6) = |6-4| = |2| = 2$.

Ex. 2 Suppose $|y-10| = x^2+1$.
(a) If we know that $y(2) = 5$, find $y(x)$.
Sol'n: We know that: when $(y-10)>0 \Rightarrow |y-10| = y-10$;
when $(y-10)<0 \Rightarrow |y-10| = -(y-10)$.
In our case, $y(2)=5$ and $(5-10)<0, \Rightarrow |y-10| = -(y-10)$.
Then $|y-10| = x^2+1 \Rightarrow -(y-10) = x^2+1 \Rightarrow$
$-y+10 = x^2+1 \Rightarrow 10-x^2-1 = y \Rightarrow \mathbf{y = 9-x^2}$.
(b) If we know that, $y(2)=15$, find $y(x)$.
In this case, $(15-10)>0 \Rightarrow |y-10| = x^2+1 \Rightarrow (y-10) = x^2+1$
$\Rightarrow y = x^2+1+10 \Rightarrow \mathbf{y = x^2+15}$.