(8) **Clarification on the DIFFERENCE between the MATRIX FORM of a l.s. and the VECTOR FORM of the SOLUTION to a l.s.**

In the past a number of students have been confused about the difference named in the title. I will illustrate it below with a specific example.

Consider a l.s.

$$\begin{aligned} x_1 + 5x_2 - 2x_3 &= 6 \\ 3x_1 + 10x_2 - x_3 &= 8. \end{aligned} \qquad (*)$$

Its MATRIX FORM is

$$\begin{pmatrix} 1 & 5 & -2 \\ 3 & 10 & -1 \end{pmatrix} \underline{x} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}, \qquad \text{with } \underline{x} \equiv \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

The solution to this l.s. is:

$$\begin{aligned} x_1 &= -4 - 3x_3 \\ x_2 &= 2 + x_3, \qquad x_3 = \text{free}. \end{aligned}$$

The VECTOR FORM of this SOLUTION is:

$$\underline{x} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} \cdot x_3$$

The VECTOR EQUATION equivalent to l.s. (*) is:

$$x_1 \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 5 \\ 10 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}.$$

(Sometimes this can be called the VECTOR FORM of the l.s. — but not "of the SOLUTION of the l.s.")