Combination of Examples 6 & 1:

How to check if a matrix is singular or nonsingular.

Check whether $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{pmatrix}$ is singular or nonsingular.

Sol'n: 1) Always start with the definition of a singular/nonsingular matrix: Find $\underline{x}$ s.t.

$$A\underline{x} = \underline{0}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{pmatrix}\underline{x} = \underline{0} \quad \Rightarrow \quad \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & -1 & 5 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right).$$

2) From this point, the work repeats that done in Step 3 of Ex. 1, and the result is:

$$\underline{x} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} x_3, \quad x_3 = \text{free}.$$

Since there is $\underline{x} \neq \underline{0}$ s.t. $A\underline{x} = \underline{0}$, $\Rightarrow$ by the definition, A is singular. //

NOTE: Even though the augmented matrix here is the same as that in Ex. 1, YOU MUST START WITH THE SETUP in STEP 1) to receive full CREDIT!