Sec. 1.8: Polynomial interpolation

We know that we need two data pts to fit a straight line, which is a polynomial of degree one.

Generalization = Thm. 14

Given $(n+1)$ points $(x_1,y_1), (x_2,y_2), \dots (x_{n+1}, y_{n+1})$ with distinct $x_1, \dots x_{n+1}$, there is a unique polynomial of degree n (or less) that passes through all these pts:

$$y = p_n(x) \equiv a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 .$$

Ex. 1 Find a quadratic polynomial

$$y = a_2 x^2 + a_1 x + a_0$$

that passes through points $(x_1,y_1), (x_2,y_2), (x_3,y_3)$.

Sol'n: Begin as in Ex. 3 of Sec. 1.3 (notes):

$@(x_1,y_1)$: $\quad x_1^2 a_2 + x_1 \cdot a_1 + 1 \cdot a_0 = y_1$

$@(x_2,y_2)$: $\quad x_2^2 a_2 + x_2 a_1 + 1 \cdot a_0 = y_2$

$@(x_3,y_3)$: $\quad x_3^2 a_2 + x_3 a_1 + 1 \cdot a_0 = y_3$

Note that here, $x_{1,2,3}$ are known, and we seek the unknown $a_{0,1,2}$.

Put the above l.s. in matrix form:

$$\begin{pmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

This l.s. is guaranteed to have a sol'n when the matrix is <u>nonsingular</u>. A matrix of the above special form is called the <u>Vandermonde</u> matrix:

$$V = \begin{pmatrix} x_1^{n-1} & \cdots & x_1 & 1 \\ x_2^{n-1} & \cdots & x_2 & 1 \\ & \vdots & & \\ x_n^{n-1} & \cdots & x_n & 1 \end{pmatrix}$$

It can be shown to be <u>nonsingular</u> (a proof for $n=3$ is in the textbook, although it is not complete).

MUST READ EX. 5 in BOOK ON YOUR OWN.

HW: 1, 5, 7, 27