<u>Ex. 6</u> Illustration for concept of <u>range</u>.

Find $\mathcal{R}(A)$, where

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

<u>Sol'n:</u> $\mathcal{R}(A)$ consists of <u>all</u> <u>possible</u> $A\underline{x}$, where $\underline{x}$ is any vector (in R^5). So:

$$A\underline{x} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \underset{\uparrow}{=} \begin{pmatrix}1\\0\\0\end{pmatrix}x_1 + \begin{pmatrix}1\\0\\0\end{pmatrix}x_2 + \begin{pmatrix}0\\1\\0\end{pmatrix}x_3 + \underbrace{\begin{pmatrix}1\\2\\0\end{pmatrix}}_{\left[\begin{pmatrix}1\\0\\0\end{pmatrix}+2\begin{pmatrix}0\\1\\0\end{pmatrix}\right]}\underline{x_4} + \underbrace{\begin{pmatrix}0\\3\\0\end{pmatrix}}_{3\begin{pmatrix}0\\1\\0\end{pmatrix}}\underline{\underline{x_5}}$$

Key Formula

$$= \begin{pmatrix}1\\0\\0\end{pmatrix}(x_1 + x_2 + \underline{x_4}) + \begin{pmatrix}0\\1\\0\end{pmatrix}(x_3 + 2\underline{x_4} + 3\underline{\underline{x_5}})$$

$$\equiv \begin{pmatrix}1\\0\\0\end{pmatrix}c_1 + \begin{pmatrix}0\\1\\0\end{pmatrix}c_2 \quad \text{(with \underline{any} } c_1, c_2\text{)}.$$

<u>Moral:</u>

- It is easy to see that $A\underline{x} = \begin{pmatrix}c_1\\c_2\\0\end{pmatrix}$, and so $\underline{y} = A\underline{x}$ will have the 3rd component (y_3) <u>equal zero</u> for any $\underline{x}$.
- Therefore, geometrically, $\mathcal{R}(A)$ will be the plane containing all vectors whose 3rd component is zero, i.e., the xy-plane in 3D.