We will show later that $y = Ce^{-3t} + 2t + 1$ is the general solution of the differential equation in Example 3. A geometric interpretation is given in Figure 1.2, which shows graphs of the general solution for representative values of C. The solution whose graph passes through the point $(t, y) = (0, 3)$ is the one that solves the initial value problem posed in Example 3.

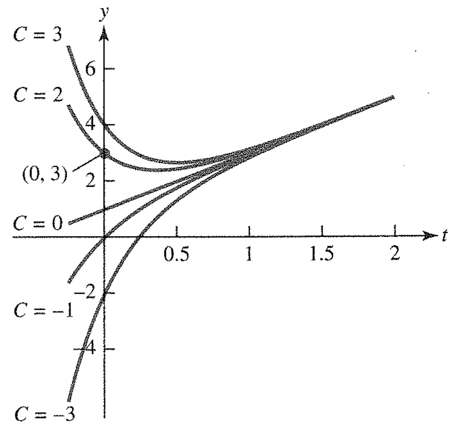

FIGURE 1.2

For any constant C, $y = Ce^{-3t} + 2t + 1$ is a solution of $y' + 3y = 6t + 5$. Solution curves are displayed for several values of C. For $C = 2$, the curve passes through the point $(t, y) = (0, 3)$; this is the solution of the initial value problem posed in Example 3.

EXERCISES

Exercises 1–4:

What is the order of the differential equation?

1. $y'' + 3ty^3 = 1$
2. $t^4y' + y\sin t = 6$
3. $(y')^3 + t^5 \sin y = y^4$
4. $(y''')^4 - \dfrac{t^2}{(y')^4 + 4} = 0$

Exercises 5–8:

For what value(s) of the constant k, if any, is $y(t)$ a solution of the given differential equation?

5. $y' + 2y = 0, \quad y(t) = e^{kt}$
6. $y'' - y = 0, \quad y(t) = e^{kt}$
7. $y' + (\sin 2t)y = 0, \quad y(t) = e^{k\cos 2t}$
8. $y' + y = 0, \quad y(t) = ke^{-t}$

9. (a) Show that $y(t) = Ce^{t^2}$ is a solution of $y' - 2ty = 0$ for any value of the constant C.

   (b) Determine the value of C needed for this solution to satisfy the initial condition $y(1) = 2$.

10. Solve the differential equation $y''' = 2$ by computing successive antiderivatives. What is the order of this differential equation? How many arbitrary constants arise in the antidifferentiation solution process?

11. (a) Show that $y(t) = C_1 \sin 2t + C_2 \cos 2t$ is a solution of the differential equation $y'' + 4y = 0$, where C_1 and C_2 are arbitrary constants.

    (b) Find values of the constants C_1 and C_2 so that the solution satisfies the initial conditions $y(\pi/4) = 3, y'(\pi/4) = -2$.

12. Suppose $y(t) = 2e^{-4t}$ is the solution of the initial value problem $y' + ky = 0, y(0) = y_0$. What are the constants k and y_0?

13. Consider $t > 0$. For what value(s) of the constant c, if any, is $y(t) = c/t$ a solution of the differential equation $y' + y^2 = 0$?

14. Let $y(t) = -e^{-t} + \sin t$ be a solution of the initial value problem $y' + y = g(t)$, $y(0) = y_0$. What must the function $g(t)$ and the constant y_0 be?

15. Consider $t > 0$. For what value(s) of the constant r, if any, is $y(t) = t^r$ a solution of the differential equation $t^2y'' - 2ty' + 2y = 0$?

16. Show that $y(t) = C_1e^{2t} + C_2e^{-2t}$ is a solution of the differential equation $y'' - 4y = 0$, where C_1 and C_2 are arbitrary constants.

Exercises 17–18:

Use the result of Exercise 16 to solve the initial value problem.

17. $y'' - 4y = 0, \quad y(0) = 2, \quad y'(0) = 0$
18. $y'' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 2$

Exercises 19–20:

Use the result of Exercise 16 to find a function $y(t)$ that satisfies the given conditions.

19. $y'' - 4y = 0, \quad y(0) = 3, \quad \lim_{t\to\infty} y(t) = 0$
20. $y'' - 4y = 0, \quad y(0) = 10, \quad \lim_{t\to-\infty} y(t) = 0$

Exercises 21–22:

The graph shows the solution of the given initial value problem. In each case, m is an integer. In Exercise 21, determine m, y_0, and $y(t)$. In Exercise 22, determine m, t_0, and $y(t)$.

21. $y'(t) = m + 1, \quad y(1) = y_0$
22. $y'(t) = mt, \quad y(t_0) = -1$

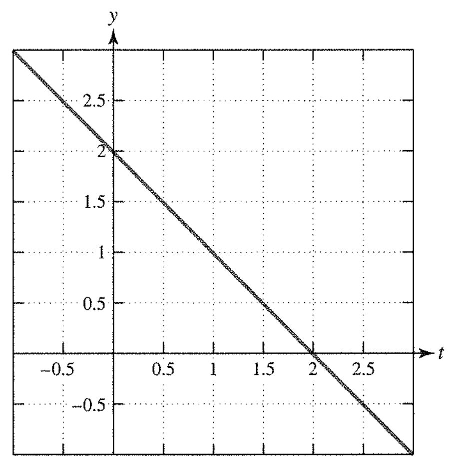

Figure for Exercise 21

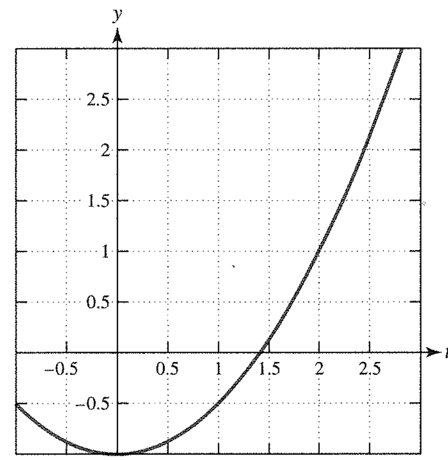

Figure for Exercise 22

23. At time $t = 0$, an object having mass m is released from rest at a height y_0 above the ground. Let g represent the (constant) gravitational acceleration. Derive an expression for the impact time (the time at which the object strikes the ground). What is the velocity with which the object strikes the ground? (Express your answers in terms of the initial height y_0 and the gravitational acceleration g.)

24. A car, initially at rest, begins moving at time $t = 0$ with a constant acceleration down a straight track. If the car achieves a speed of 60 mph (88 ft/sec) at time $t = 8$ sec, what is the car's acceleration? How far down the track will the car have traveled when its speed reaches 60 mph?