CALCULUS II, MATH 022 — Instructor: Dr. T.I. Lakoba

Strategy of testing a series for convergence

1. If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges (the Divergence Test).
   Note that if $\lim_{n\to\infty} a_n = 0$, this alone does not mean that the series converges.
2. If the series is of the form $\sum \frac{1}{n^p}$, it is a p-series. It converges for $p > 1$ and diverges for $p \le 1$.
3. If the series has (or can be brought to) the form $\sum b \cdot r^n$, it is a geometric series. It converges for $|r| < 1$ and diverges for $|r| \ge 1$.
4. If $a_n = f(n)$, where $\int_1^\infty f(x)dx$ is easily evaluated, then the Integral Test can (and should) be used.
5. If terms of the series *are all positive* and look similar (at least for large n) to terms of a geometric series or a p-series, then the (Limit) Comparison Test can (and should) be used.
6. If the series has the form $\sum(-1)^{n+1}a_n$ with $a_n \ge 0$, then the Alternating Series Test or the Divergence Test should be used.
7. If the series involves factorials and products, then the Ratio test can be used. Namely, if

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \text{ is } \begin{cases} <1 \\ >1 \\ =1 \end{cases}, \quad \text{then the series } \begin{cases} \text{converges absolutely} \\ \text{diverges} \\ \textit{no information} \end{cases}.$$

8. Finally, suppose that a series is represented as a sum or difference of two simpler series.
   - If both of these series converge, then the original series converges.
   - If one of these series converges and the other one diverges, then the original series diverges.
   - The sum of two *positive* divergent series is divergent.
   - The difference of two *positive* divergent series is indeterminate (i.e. may converge or may diverge).

In the examples below, only the method to be used is indicated.

EX. 1

$$\sum \frac{n-1}{n^2+n}$$

For very large n, this looks like a p-series: $a_n \approx \frac{n}{n^2} = \frac{1}{n}$.

Use the Limit Comparison test and then compare what you got with a p-series.

EX. 2

$$\sum (-1)^{n+1}\frac{n-1}{n^2+n}$$

Use either the Alternating Series Test or the Divergence Test.

EX 3 $\sum \frac{10^n}{n!}$

Use the Ratio Test.

EX 4 $\sum n^2 e^{-n^3}$

Since one can evaluate $\int_1^\infty x^2 e^{-x^3}dx$ by a u-substitution, use the Integral Test.

EX 5 $\sum \sin n$

Use the Divergence Test.

EX 6 $\sum \frac{(-1)^n}{\sqrt{3n-1}}$

Use the Alternating Series Test or the Divergence Test.

EX 7 $\sum \frac{3^n}{2^n+n}$

Use the Limit Comparison Test (compare with a geometric series $\sum \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n$).

EX 8 $\frac{\sqrt{n^2-1}}{3n^3+5}$

Use the Limit Comparison Test (compare with a p-series).

EX 9 $\sum \frac{n^2+5}{3^n}$

Since this involves an exponential (3^n), then the Ratio Test should be used.

$\sum_{n=1}^\infty (-1)^n \frac{1}{n^p}$ — If $p = 0$, $\sum_{n=1}^\infty (-1)^n$ div. If $p < 0$, $\sum_{n=1}^\infty (-1)^n n^{-p}$ div. If $p > 0$, $\lim_{n\to\infty}\frac{1}{n^p} = 0$

$\sum_{n=1}^\infty \frac{1}{3n^2+4n+5} \Rightarrow \sum_{n=1}^\infty \frac{1}{n^2} \Rightarrow$ both converge

$\sum_{n=1}^\infty \frac{1}{\sqrt{3n-2}} \Rightarrow \sum_{n=1}^\infty \frac{1}{\sqrt{n}} \Rightarrow$ both diverge

$\sum_{n=1}^\infty \frac{n^2+10}{4n^5+n^3} \Rightarrow \sum_{n=1}^\infty \frac{n^2}{n^5} \Rightarrow \frac{1}{n^3}$ both converge

$\sum_{n=1}^\infty \frac{\arctan n}{n^2+1}$ $f'(x) = \frac{1-2x\arctan x}{(x^2+1)^2}$ $\int_1^\infty \frac{\arctan x}{x^2+1}dx = \left[\frac{(\arctan x)^2}{2}\right]_1^\infty = \frac{3\pi^2}{32}$

Direct Comparison test – Let $0 < a_n \le b_n$ for all n

If $\sum_{n=1}^\infty b_n$ converges, then a_n converges

If $\sum_{n=1}^\infty a_n$ diverges, then b_n diverges

Taylor $P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$

Maclaurin $(c=0)$ $P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n$

$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$

$\sum_{n=2}^\infty \frac{1}{n\ln n}$ $\int_2^\infty \frac{1}{x\ln x}dx = \lim_{b\to\infty}\left[\ln(\ln x)\right]_2^b = \infty$ $f'(x) = -\frac{1+\ln x}{x^2(\ln x)^2}$

$\int (\ln x)dx = x\ln x - x$

Partial Fraction $\int \frac{x}{(x^2-1)} = \int \frac{x}{(x-1)(x+1)(x^2+1)}$ $= \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

$\int \sin^5 x \cos^2 x \, dx = \int \sin x(1-\cos^2 x)^2 \cos^2 x \, dx$ $= -\int (\cos^2 x - 2\cos^4 x + \cos^6 x)(-\sin x)dx$

$u = \cos x$, $du = -\sin x \, dx$

Test	Series	Converge	Diverge	Extra
nth term	$\sum_{n=1}^\infty a_n$		$\lim_{n\to\infty} a_n \ne 0$	not used to show convergence
Geometric series	$\sum_{n=0}^\infty ar^n$	$\|r\| < 1$	$\|r\| \ge 1$	sum $= \frac{a}{1-r}$
Telescoping series	$\sum_{n=1}^\infty (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		sum $= b_1 - L$
p-series	$\sum_{n=1}^\infty \frac{1}{n^p}$	$p > 1$	$p \le 1$	
Alternating series	$\sum_{n=1}^\infty (-1)^{n-1} a_n$	① $0 < a_{n+1} \le a_n$ ② $\lim_{n\to\infty} a_n = 0$		Remainder $\|R_n\| \le a_{n+1}$
Integral (f is continuous, positive, decreasing)	$\sum_{n=1}^\infty a_n$, $a_n = f(n) \ge 0$	$\int_1^\infty f(x)dx$ converges	$\int_1^\infty f(x)dx$ diverges	Remainder $0 < R_N < \int_N^\infty f(x)dx$
Root	$\sum_{n=1}^\infty a_n$	$\lim_{n\to\infty}\sqrt[n]{\|a_n\|} < 1$	$\lim_{n\to\infty}\sqrt[n]{\|a_n\|} > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty}\sqrt[n]{\|a_n\|} = 1$
Ratio	$\sum_{n=1}^\infty a_n$	$\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| < 1$ (Converges absolutely)	$\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| > 1$ or $= \infty$	Inconclusive if $\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| = 1$
Direct comparison ($a_n, b_n > 0$)	$\sum_{n=1}^\infty a_n$	$0 < a_n \le b_n$ and $\sum_{n=1}^\infty b_n$ converges	$0 < b_n \le a_n$ and $\sum_{n=1}^\infty b_n$ diverges	
Limit comparison ($a_n, b_n > 0$)	$\sum_{n=1}^\infty a_n$	$\lim_{n\to\infty}\frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^\infty b_n$ converges	$\lim_{n\to\infty}\frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^\infty b_n$ diverges	

Power Series $\sum a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$

Power Series centered at c: $\sum a_n(x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \dots + a_n(x-c)^n$

① Series converges at/only c

② $R > 0$, such that the series converges absolutely for $|x-c| < R$ and diverges for $|x-c| > R$

③ If a series converges at c (only), RoC $= 0$

If a series converges for all x, RoC $= \infty$

Functions of a power series: $f'(x) = \sum_{n=1}^\infty n a_n (x-c)^{n-1}$

$\int f(x)dx = C + \sum_{n=0}^\infty a_n \frac{(x-c)^{n+1}}{n+1} = C + a_0(x-c) + a_1\frac{(x-c)^2}{2} + a_2\frac{(x-c)^3}{3}$

Absolute Value Theorem: $\lim |a_n| = 0$ then $\lim a_n = 0$

$\lim_{n\to\infty}\frac{1}{n!} = 0$

...ple Square → $x^2 + 10x$; $a^2 + 2\cdot b\cdot a + b^2 \rightarrow (a+b)^2 - b^2$

$\int u\,dv = uv - \int v\,du$ | $\int \cos = \sin$

$\frac{(n+1)!}{n!} = \frac{n!(n+1)}{n!} = n+1$ | $\frac{1/x}{1/(x-1)^2} = \frac{(x-1)^2}{x}$

$(A+B)^2 = A^2 + 2AB + B^2$
$(A-B)^2 = A^2 - 2AB + B^2$ | $\int \frac{dy}{y} = \ln|y|$

$\int f(x)g(x)\,dx \neq \int f(x)\,dx - \int g(x)\,dx$

$f(x) = \ln x$, $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$, $f'''(x) = \frac{2}{x^3}$

$5 = \ln(2.5)$, $K = \frac{\ln(2.5)}{5}$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$, $\sin^2 u = \frac{1-\cos 2u}{2}$, $\cos^2 u = \frac{1+\cos 2u}{2}$

$\sin 2u = 2\sin u\cos u$
$\cos 2u = 1 - 2\sin^2 u$; $\tan 2u = \frac{2\tan u}{1-\tan^2 u}$

$\frac{\pi}{2} = 90°$, $\pi = 180°$, $270° = \frac{3\pi}{2}$, $360° = 2\pi$

$30° = \frac{\pi}{6}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $45° = \frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$60° = \frac{\pi}{3} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$(u^n)' = nu^{n-1}u'$, $(e^u)' = e^u u'$

$(\sin x)' = (\cos x)x'$ | $(\cos x)' = -(\sin x)x'$
$(\tan x)' = (\sec^2 x)x'$ | $(\cot x)' = -(\csc^2 x)x'$
$(\sec x)' = (\sec x\tan x)x'$ | $(\csc x)' = -(\csc x\cot x)x'$

$(\arcsin x)' = \frac{x'}{\sqrt{1-x^2}}$ | $(\arccos x)' = \frac{-x'}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{x'}{1+x^2}$ | $(\text{arc}\cot x)' = \frac{-u'}{1+u^2}$

$(\text{arc}\csc x)' = \frac{-x'}{|x|\sqrt{x^2-1}}$ | $(\text{arc}\sec x)' = \frac{u'}{|x|\sqrt{x^2-1}}$

$x^2 - x$

$x^2 - 2\cdot\frac{1}{2}\cdot x + \frac{1}{4} - 1$

$\frac{1}{2x}x$; $\frac{1}{2x}\cdot\frac{2x}{1}$

$\frac{1}{0} = \infty$, $\frac{1}{\infty} = 0$, $\ln 1 = 0$, $\ln\infty = \infty$, $x\ln x = \frac{\ln x}{1/x}$

$\frac{\infty}{e^\infty} = 0$, $\int\sin = -\cos$, $\int\sec^2 = \tan$, $\int\sec\tan = \sec$, $\int\csc^2 = -\cot$

$\int\csc\cot = -\csc$, $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$, | $\ln L = K \Rightarrow L = e^K$

$\ln a^b = b\cdot\ln a$, $(\ln x)' = \frac{1}{x}dx$, $(t^n)' = nt^{n-1}$

$(a^x)' = \ln a\cdot a^x$, $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$, $x^a\cdot x^b = x^{a+b}$

$\lim_{x\to\infty}\frac{1+\cos x}{1} \Rightarrow$ DNE, $\int\frac{dx}{x} = -\ln|x| + C$

$\int\frac{dx}{\sqrt{x}} = \frac{x^{1/2}}{1/2} + C$, $\int\frac{du}{\sqrt{1-u^2}} = \arcsin u + C$

$\int\frac{dx}{1+\sin x} = \frac{\sin x\cdot 1}{\cos x} + C$, $\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C$

$\int_1^\infty\frac{dx}{x^p}$ con → $p>1$, div → $p\le 1$; Area of a Circle $= \int_{-a}^{a} 2\sqrt{a^2-x^2}\,dx$ | $x = a\sin\theta$, $a^2 - x^2 = a^2 - a^2\sin^2\theta$

$\int e^{-x/2} = -2e^{-x/2}$, $\int 2x^{-1/2}\,dx = 2x^{1/2}$

$\sin\infty = [-1,1]$, $\cos 0 = 1$, $\sin^2\theta = 1 - \cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$, $\cosh^2 t - \sinh^2 t = 1$

$\sin^2 x = \frac{1-\cos(2x)}{2}$, $\cos^2 x = \frac{1+\cos(2x)}{2}$

$\sin x = \frac{2\tan x/2}{1+\tan^2\frac{x}{2}}$, $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$; $u = \tan\frac{x}{2}$, $x = 2\arctan u$, $dx = 2\cdot\frac{1}{1+u^2}du$

$\int\frac{dx}{\sqrt{1+x^2}} = \int\sec\theta\,d\theta$ when $x = \tan\theta$

$\int e^u\,du = e^u + C$, $\int\csc x\cot x = -\csc x$

$\int\frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

$\int\frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$\int\frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arc}\sec\frac{|u|}{a} + C$

$f(x) = \cos x$, $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$
$P_4(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$
$f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$
$c = \pi/6$
$P_3(x) = f\left(\frac{\pi}{6}\right) + f'\left(\frac{\pi}{6}\right)\left(x-\frac{\pi}{6}\right) + \frac{f''\left(\frac{\pi}{6}\right)}{2!}\left(x-\frac{\pi}{6}\right)^2 + \frac{f'''\left(\frac{\pi}{6}\right)}{3!}\left(x-\frac{\pi}{6}\right)^3$

$f(x) = e^x$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots \frac{x^n}{n!}$

$[-1, 1) \longrightarrow$ Converges for -1 / diverges for 1

$(x^2 - x + 1)$; $(x(x-1) + 1)$

$(x\quad)(x\quad)$

$\frac{dy}{dx} + y\left(\frac{1}{2x}\right)$

1. $\int \sin^{2k+1} x \cos^n x\, dx = \int (\sin^2 x)^k \cos^n x \sin x\, dx = \int (1-\cos^2 x)^k \cos^n x \sin x\, dx$

2. $\int \sin^m x \cos^{2k+1} x\, dx = \int \sin^m (\cos^2 x)^k \cos x\, dx = \int \sin^m x (1-\sin^2 x)^k \cos\, dx$

Wallis formula $n = \text{odd}\ (n \geq 3)$ $\int_0^{\pi/2} \cos^n x\, dx = \left(\frac{n-1}{n}\right)$ / $n = \text{even}$, $n \geq 2$ $\int_0^{\pi/2} \cos^n x\, dx = \left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right)$

sec = even

1. $\int \sec^{2k} x \tan^n x\, dx = \int (\sec^2 x)^{k-1} \tan^n x \sec^2 x\, dx = \int (1+\tan^2 x)^{k-1} \tan^n x \sec^2 x\, dx$

tan = odd

2. $\int \sec^m x \tan^{2k+1} x\, dx = \int \sec^{m-1} x (\tan^2 x)^k \sec x \tan x\, dx = \int \sec^{m-1} x (\sec^2 x - 1)^k \sec x \tan x\, dx$

tan = even / no sec

3. $\int \tan^n x\, dx = \int \tan^{n-2} x (\tan^2 x)\, dx = \int \tan^{n-2} x (\sec^2 x - 1)\, dx$

Diff angles

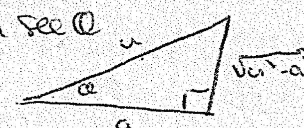
$\sin mx \sin nx = \frac{1}{2}(\cos(m-n)x) - \cos(m+n)x$

$\sin mx \cos nx = \frac{1}{2}(\sin(m-n)x) + \sin(m+n)x$

$\cos mx \cos nx = \frac{1}{2}(\cos(m-n)x) + \cos(m+n)x$

3 - $\sqrt{u^2-a^2} = \pm a \tan\theta$ + - $u > a$ - - $u < -a$

$u = a\sec\theta$



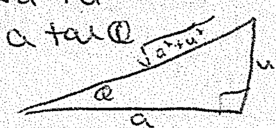
Trig substitution

1 - $\sqrt{a^2-u^2} = a\cos\theta$

Let $u = a\sin\theta$

2 - $\sqrt{a^2+u^2} = a\sec\theta$

$u = a\tan\theta$



rational f(x) sine + cosine

$u = \frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

$\cos x = \frac{1-u^2}{1+u^2}$, $\sin x = \frac{2u}{1+u^2}$, $dx = \frac{2du}{1+u^2}$

Special Integ. $a > 0$

1 - $\sqrt{a^2-u^2}\, du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2-u^2}\right) + C$

2 - $\sqrt{u^2-a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2-a^2} - a^2 \ln|u+\sqrt{u^2-a^2}|\right) + C$, $u > a$

3 - $\sqrt{u^2+a^2}\, du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2 \ln|u+\sqrt{u^2+a^2}|\right) + C$

3 - $\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{c} f(x)\, dx + \int_{c}^{\infty} f(x)$

if f is cont. on $(-\infty, \infty)$

Improper Integrals with improper limits

1 - $\int_a^{\infty} f(x)\, dx = \lim_{b\to\infty} \int_a^b f(x)\, dx$

f is cont. on $[a, \infty)$

f is cont on $(-\infty, b]$

2 - $\int_{-\infty}^{b} f(x)\, dx = \lim_{a\to-\infty} \int_a^b f(x)\, dx$

$[a, b]$ disc c in (a, b)

3 - $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)$

Improper Integrals with Infinite Disc.

1 - $\int_a^b f(x)\, dx = \lim_{c\to b^-} \int_a^c f(x)\, dx$

Interval $[a, b)$, Disc at b

Int $(a, b]$ disc at a

2 - $\int_a^b f(x)\, dx = \lim_{c\to a^+} \int_c^b f(x)\, dx$

$\int C \cdot f(x)\, dx = C \cdot \int f(x)\, dx$

$\int u^n e^u\, du = u^n e^u - n\int u^{n-1} e^u\, du$

$\int u e^u\, du = (u-1)e^u + C$

$\int e^{-2x}\, dx = \frac{1}{-2e^{2x}}$

$\int e^{au} \sin bu\, du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

$\int e^{au} \cos bu\, du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

$\int e^u \frac{du}{a} = \frac{1}{a}\int e^u\, du$

$\int \frac{1}{1+e^u}\, du = u - \ln(1+e^u) + C$

$(4)^{1/2} = \sqrt{4}$

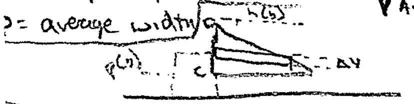
$(4)^{3/2}$ $\sqrt{\ }$

Area of a region btw f(x) + g(x) → $\int_a^b f(x) - g(x)\, dx$

Disk - Horizontal Axis of Rev → $V = \pi\int_a^b (R(x)^2)\, dx$
Vertical Axis of Rev → $V = \pi\int_c^d (R(y)^2)\, dy$

Washer - Outer radius - $R(x)$, inner radius - $r(x)$: $V = \pi\int_a^b (R(x)^2) - (r(x)^2)\, dx$

Shell - $p + w/2$ = outer radius, $p - w/2$ = inner radius, w = average width
H Axis of R → $V = 2\pi\int_c^d (p(y)\, h(y))\, dy$
V Axis of R → $V = 2\pi\int_a^b (p(x)\, h(x))\, dx$



Distance → $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
Def of arc length → $S = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$
Def of surface revolution → $S = 2\pi\int_a^b r(x)\sqrt{1+f'(x)^2}\, dx$
$r(x)$ = Distance btwn Line + axis of rev

$W = Fd$ Def of work → $\int_a^b F(x)\, dx$ ($a = L_{initial} - L_{natural}$, $b = L_{final} - L_{natural}$)
$F = (d)$(spring constant) $f(x) = \frac{c}{x^2} = (e)$ extra
$F = (\text{constant})\left(\frac{m_1 m_2}{(d)^2}\right)$
$F = k \cdot \frac{q_1 q_2}{(d)^2} = k \cdot \frac{(\text{charge})(\text{charge})}{d^2}$ $\Delta F = w(\pi x^2 \Delta y)$, $\Delta W = \Delta F (x)$

moment = (mass)(length of arm) $M_y = m_1x_1 + m_2x_2 \ldots$ $F = ma$
M_0 → origin → $m_1x_1 + m_2x_2 + m_3x_3 = 0$ $M_x = m_1y_1 + m_2y_2 \ldots$
$\bar{x} = \frac{M_y}{m}$ $\bar{y} = \frac{M_x}{m}$ ρ = density

Moments + Centers of Mass of a Planar Lamina
$M_x = \rho\int_a^b \left(\frac{f(x)+g(x)}{2}\right)(f(x) - g(x))\, dx$
$M_y = \rho\int_a^b x[f(x) - g(x)]\, dx$

Mass of lamina → $\rho\int_a^b f(x) - g(x)\, dx$
Pappus → $V = 2\pi r A = 2\pi$(d btw centroid + line)(Area of R)

Exp growth + Model Decay → $y = Ce^{kt}$
C = initial value of y, k = proportionality constant; $k > 0$ → Exp growth; $k < 0$ → model decay

Homogeneous Diff eq → $f(tx, ty) = t^n f(x,y)$
$M(x,y)\,dx + N(x,y)\,dy = 0$
Let $y = vx$ to change variables

Chemical Rxn → $\frac{dy}{dt} = ky^2$

Pop growth $\frac{dN}{dt} = k(650 - N) \Rightarrow N = 650 - Ce^{-kt}$
$\frac{dy}{dt} = ky \ln\frac{(\text{max pop})}{y}$

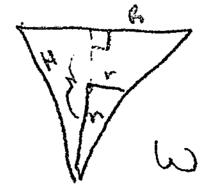
Logistic Diff eq → $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$
L = carrying capacity
$\left|\frac{L-y}{y}\right| = e^{-kt-C}$ $\frac{L-y}{y} = be^{-kt}$ let $b = \pm e^{-C}$

Point of Inflection: $y'' > 0$ concave upward; $y'' < 0$ concave downward

First order Diff eq → $y' + P(x)y = Q(x)$
Diff eq ⇒ $ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}\, dx + C$
$xy' - 2y = x^2$ $P(x) = -\frac{2}{x}$ $\int P(x)dx = -\int \frac{2}{x}dx$
$y' + P(x)y = Q(x)$ $= \frac{1}{e^{2\ln x}} = \frac{1}{x^2}$
$y' - \left(\frac{2}{x}\right)y = x$ x on each side

Bernoulli Eq → $y' + P(x)y = Q(x)y^n$
$y^{1-n} e^{\int(1-n)P(x)dx} = \int (1-n)Q(x)e^{\int(1-n)P(x)dx}\, dx + C$

Bucket of water problem → $W = \int_0^H (Mg + \rho g(H-x))$
M rope = L rope · ρ
L rope = $(H - x)\rho$



$\frac{R}{H} = \frac{r}{h} \Rightarrow r = x \cdot \frac{R}{H}$
$W = \int_a^b \rho g A_{base}(x) \cdot (H - x)\, dx$
$= \frac{1}{3}\pi\rho g\left(\frac{B}{H}\right)^2 \cdot h^3 \cdot \left(H - \frac{3}{4}h\right)$

ODE $\left(\frac{dy}{dx} = \frac{2y}{x}\right) = \left(\frac{1}{y}dy = \frac{2dx}{x}\right) = \left(\int\frac{dy}{y} = 2\int\frac{dx}{x}\right)$

$f(x) = \frac{1}{2-x}$, $c = 5$ ⇒ $\frac{a}{1-r} = \frac{1}{-3-(x-5)}$
$= \frac{-1/3}{1+(1/3)(x-5)}$ $a = -1/3$ $r = (-1/3)(x-5)$ (Ex)
$\frac{1}{2-x} = \sum_{n=0}^{\infty} -\frac{1}{3}\left[-\frac{1}{3}(x-5)\right]^n = \frac{(x-5)^n}{(-3)^{n+1}}$

$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$
$f(x) = \ln(x+1) = \int \frac{1}{1+x}\, dx$
$\ln(x+1) = \int\left[\sum_{n=0}^{\infty} (-1)^n x^n\right] dx = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$
$-1 < x \le 1$

$\int \frac{1}{2}x$ $\frac{dy}{dx} = -\frac{y}{2x}$
$\frac{2x}{dx} = -y/dy$