<u>Ex. 6 (a)</u> Find <u>the form</u> of y_p for:

$$y'' - y' = t(e^t - 2) \equiv g(t)$$

<u>Solution:</u> $g(t) = g_1 + g_2$, where

$g_1 = te^t$, $g_2 = -2t$. So $y_p \equiv y_{p1} + y_{p2}$, where

y_{p1} corresponds to g_1, y_{p2} corresponds to g_2.

1) Find y_h: $y = e^{\lambda t} \to y_h'' - y_h' - 0 \Rightarrow$

$\lambda^2 - \lambda = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = 0$. $\boxed{y_h = c_1 e^t + c_2}$

(since $e^{0 \cdot t} = 1$).

2) Find (y_{p1}) for $g_1 = te^t$.

According to the Table on p. 163 (see also our Ex. 2), the form of y_p is:

$$t^r (A_1 t + A_0) e^t, \qquad (\bigstar)$$

where r is chosen so that <u>no part of</u> ($\bigstar$) <u>is also part of y_h.</u>

<u>$r = 0$</u> does not work, since $A_0 e^t$ is part of y_h.

<u>$r = 1$</u> <u>does</u> work, since neither $A_1 t^2 e^t$ nor $A_0 t e^t$ is part of y_h.

So, the <u>form</u> of $y_{p1} = t(A_1 t + A_0)e^t \equiv (A_1 t^2 + A_0 t)e^t$.

A_1, A_0 can be found as in Ex. 2, 4.

3) Find (y_{p2}) corresponding to $g_2 = -2t$

According to the Table on p. 163 and our Ex. 2:

$$y_{p2} = t^r \cdot (B_1 t + B_0), \qquad (\star\star)$$

where r is chosen so that no part of $(\star\star)$ is also part of y_h.

$r=0$ does not work, since then B_0 is part of y_h.

$r=1$ does work since neither $B_1 t^2$ nor $B_0 t$ is part of y_h.

So, the form of $y_{p2} = t(B_1 t + B_0) \equiv B_1 t^2 + B_0 t$.

Finally, recall that $y_p = y_{p1} + y_{p2}$.

Ex. 6(b) (in some sense, inverse to Ex. 6(a)).

Let the DE be $y'' + \alpha y' + \beta y = g(t)$ with $\alpha, \beta = \text{const}$, $g(t) = te^t - 2t + e^{3t}$. If the form

$$y_p = (a_2 t^2 + a_1 t)e^t + (b_2 t^2 + b_1 t) + ce^{3t},$$

find α, β.

Sol'n: 1) The term $g_1 = te^t$ gave rise to $y_{p1} = (a_2 t^2 + a_1 t)e^t = t(a_2 t + a_1)e^t$.

According to the Table on p.163, that g_1 gives rise to: $(y_{p1})_{\text{Table}} = t^r \cdot (A_1 t + A_0)e^t$.

Comparing the given and the Table's (y_{p1}), we see that $r=1$ and hence $r=0$ would not work.

"$r=0$ does not work" $\Rightarrow$ One of $\{te^t, e^t\}$ is part of y_h.

"$r=1$ works" $\Rightarrow$ Neither of $\{t^2e^t, te^t\}$ is part of y_h.

Comparing the above statements for $r=0$ & $r=1$, we conclude that $y_h = e^t = e^{1\cdot t}$, $\Rightarrow \lambda_1 = 1$

2) The work for $g_2 = -2t$ follows the same steps. This g_2 gave rise to $y_{p2} = (b_2t^2 + b_1t) = t\cdot(b_2t + b_1)$. According to the Table, it should give rise to $(y_{p2})_{Table} = t^r\cdot(B_1t + B_0)$.

Comparing the given and Table's y_{p2}, we see that $r=1$, which implies that $r=0$ would not have worked.

"$r=0$ does not work" $\Rightarrow$ One of $\{t, 1\}$ is part of y_h.

"$r=1$ works" $\Rightarrow$ Neither of $\{t^2, t\}$ is part of y_h.

Comparing the above statements for $r=0$ & $r=1$, we conclude that $y_h = 1 = e^{0\cdot t}$, $\Rightarrow \lambda_2 = 0$.

3) $g_3 = e^{3t}$ gave rise to ce^{3t}, $\Rightarrow$ according to the Table on p. 163, e^{3t} is not part of y_h (i.e. $\lambda \neq 3$). Actually, we already know this, since a 2nd-order DE can only have 2 roots, and we have already found both of them: $\lambda_1 = 1$, $\lambda_2 = 0$.

4) Finally, find α, β.

$(\lambda-\lambda_1)(\lambda-\lambda_2) = (\lambda-1)(\lambda-0) = \lambda^2-\lambda \Rightarrow$ the lhs. of the DE is $y'' \underbrace{-y'}_{\alpha y'} + \underbrace{0\cdot y}_{\beta y}$, $\Rightarrow$

$\alpha=-1, \ \beta=0$.