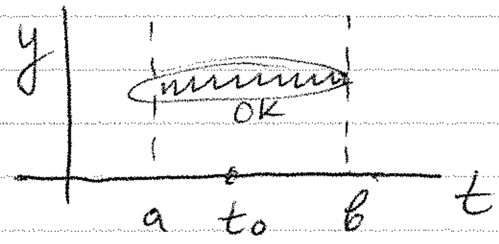
Lecture 6 Existence & uniqueness of solution of nonlinear DEs

Recall Thm. 2.1 for linear DEs:

$$y' + p(t)\,y = g(t)$$
$$y(t_0) = y_0$$

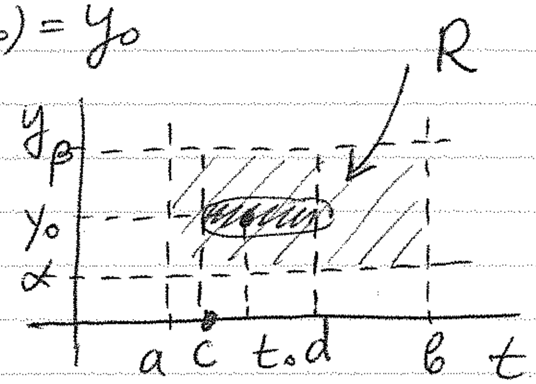


$$\begin{pmatrix} p(t),\ g(t) \\ \text{continuous} \\ \text{on } (a,b) \end{pmatrix} \Rightarrow \begin{pmatrix} y(t) \\ \text{exists \&} \\ \text{is unique on } (a,b) \end{pmatrix}$$

Thm. 2.2 (for nonlinear DEs)

$$y' = f(t,y), \quad y(t_0) = y_0$$

- $f(t,y)$ & $\frac{\partial f(t,y)}{\partial y}$ are functions of (t,y) $\Rightarrow$ they are defined in the (t,y)-plane.



- Suppose f & f_y are continuous in some $R = (a,b) \times (\alpha, \beta)$ in (t,y)-plane, and let $(t_0, y_0) \in R$.

Then $y(t)$ is guaranteed to exist for some $t \in (c,d)$, where (c,d) is inside (a,b). However, we cannot predict how wide (c,d) is.

<u>Note:</u> This should be contrasted with the situation for linear DEs: There, if $f(t,y) = -p(t)y + g(t)$ and $f_y = -p(t)$ are continuous on $t \in (a,b)$, then the solution is guaranteed to exist on the <u>entire</u> (a,b).

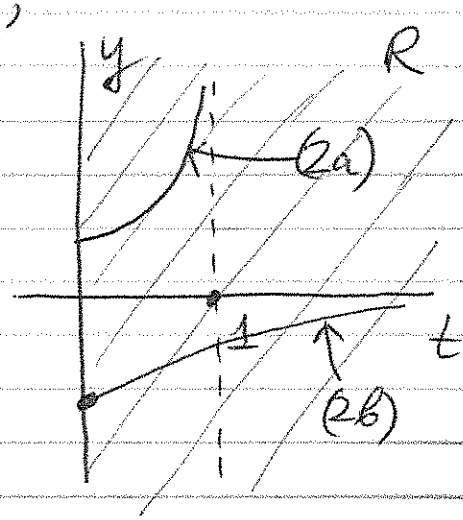
2/5/16

An example where (c,d) may or may not be the entire (a,b) for a nonlinear DE was illustrated by Ex. 2(a,b) of Lecture 5.

<u>Recall:</u>

<u>Ex. 2</u> $y' = y^2 \Rightarrow f(t,y) = y^2$; $f_y(t,y) = 2y$

are continuous on $(t,y) \in (0,\infty) \times (-\infty,\infty)$



(2a) $\underline{y(0)=1} \Rightarrow y = \frac{1}{1-t}$

$\Rightarrow$ blows up @ $t=1$

$\Rightarrow (c,d) = (0,1)$ while $(a,b) = (0,\infty)$.

2(b) $\underline{y(0)=-1} \Rightarrow y = \frac{-1}{1+t} \Rightarrow$ exists for $t \in (0,\infty)$

$\Rightarrow (c,d) = (0,\infty) \leftarrow$ same as (a,b).

<u>So, to repeat</u>:

All that the Thm. for nonlinear eqs. is able to say is that the solution of the IVP exists only <u>sufficiently near</u> the initial point $t=t_0$.

<u>HW</u>: Sec. 2.5 # 5, 9.