It's clear that a guess of the form

$$y_P(t) = Ae^{-t}$$

will not be the appropriate form for a particular solution, because e^{-t} is a solution of the homogeneous equation. A guess of the form $y_P(t) = Ate^{-t}$ will fail for the same reason. It's perhaps not surprising that a guess of the form

$$y_P(t) = At^2e^{-t}$$

does work. Substituting this form of the trial solution leads to

$$(At^2e^{-t} - 4Ate^{-t} + 2Ae^{-t}) + 2(-At^2e^{-t} + 2Ate^{-t}) + At^2e^{-t} = 2e^{-t}.$$

Simplifying, we obtain

$$2Ae^{-t} = 2e^{-t},$$

so $2A = 2$ and hence $y_P(t) = t^2e^{-t}$. The general solution is

$$y(t) = c_1e^{-t} + c_2te^{-t} + t^2e^{-t}.$$ ❖

A Table Summarizing the Method of Undetermined Coefficients

We summarize the method of undetermined coefficients in Table 3.1. The method applies to the nonhomogeneous linear differential equation

$$ay'' + by' + cy = g(t),$$

where a, b, and c are constants and $g(t)$ has one of the forms listed on the left-hand side of the table. The corresponding form to assume for the particular solution is listed on the right-hand side of the table. The forms listed in Table 3.1 will work; that is, they will always yield a particular solution. In the Exercises, we ask you to solve problems using Table 3.1. The role of the factor t^r in the

TABLE 3.1

The right-hand column gives the proper form to assume for a particular solution of $ay'' + by' + cy = g(t)$. In the right-hand column, choose r to be the smallest nonnegative integer such that no term in the assumed form is a solution of the homogeneous equation $ay'' + by' + cy = 0$. The value of r will be 0, 1, or 2.

Form of $g(t)$	Form to Assume for a Particular Solution $y_P(t)$
$a_nt^n + \cdots + a_1t + a_0$	$t^r[A_nt^n + \cdots + A_1t + A_0]$
$[a_nt^n + \cdots + a_1t + a_0]e^{\alpha t}$	$t^r[A_nt^n + \cdots + A_1t + A_0]e^{\alpha t}$
$[a_nt^n + \cdots + a_1t + a_0]\sin\beta t$ or $[a_nt^n + \cdots + a_1t + a_0]\cos\beta t$	$t^r[(A_nt^n + \cdots + A_1t + A_0)\sin\beta t + (B_nt^n + \cdots + B_1t + B_0)\cos\beta t]$
$e^{\alpha t}\sin\beta t$ or $e^{\alpha t}\cos\beta t$	$t^r[Ae^{\alpha t}\sin\beta t + Be^{\alpha t}\cos\beta t]$
$e^{\alpha t}[a_nt^n + \cdots + a_0]\sin\beta t$ or $e^{\alpha t}[a_nt^n + \cdots + a_0]\cos\beta t$	$t^r[(A_nt^n + \cdots + A_0)e^{\alpha t}\sin\beta t + (B_nt^n + \cdots + B_0)e^{\alpha t}\cos\beta t]$