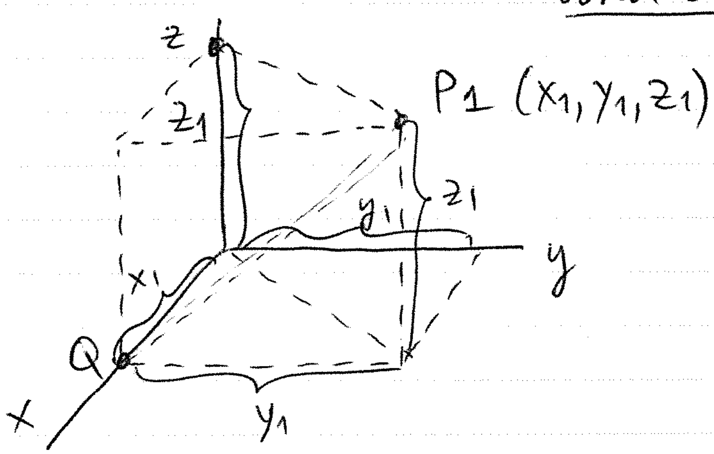
Calculus III

J. Stewart, 8th Ed. (2017), Early Transcendentals

Lecture 1: Review of coordinates & vectors

① Coordinates
(Sec. 12.1)

a Definition, distance to coord. planes and coord. axes



distance from P_1 to xy-plane: z_1;

dist. from P_1 to x-axis:

$|P_1Q| = \sqrt{y_1^2 + z_1^2}$.

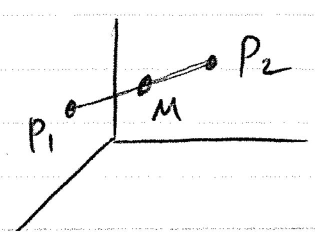
etc.

b Planes ∥ to coord. planes

$x = x_1 \leftarrow$ plane ∥ zy-plane, x_1 units away from it (contains Q, P_1 in the figure).

c Midpoint of segment

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right).$$

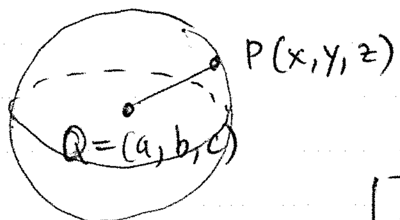


d. **Distance between points:**

$$|P_1P_2| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

(like the Pythagorean Thm. in 3D).

e. **Sphere**: all points equidistant from center $Q(a,b,c)$:



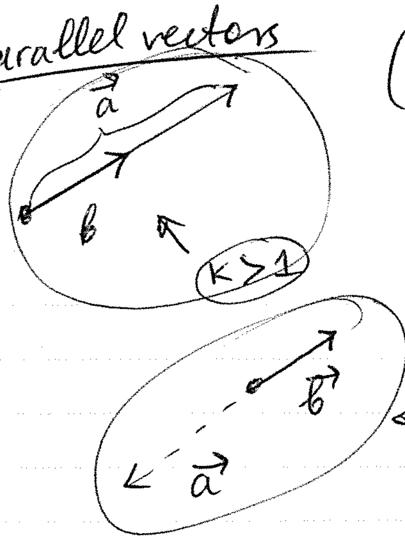
$|PQ| = R$, or

$$\boxed{\sqrt{(x-a)^2+(y-b)^2+(z-c)^2} = R}$$

2. Vectors (Sec. 12.2)

a. **Parallel vectors**

$(\vec{a} \parallel \vec{b}) \Leftrightarrow (\vec{a} = k \cdot \vec{b})$ for some scalar k.



$$k = \frac{\text{length } \vec{a}}{\text{length } \vec{b}} = \frac{|\vec{a}|}{|\vec{b}|}.$$

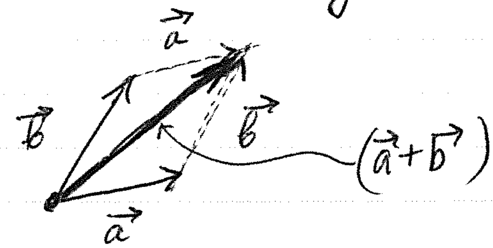
$k<0$.

b. **Sum & difference of vectors**

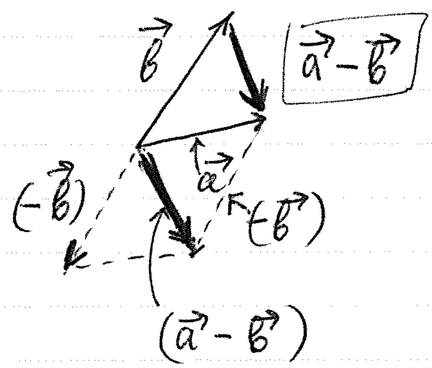
Algebraically:

$$\vec{a} \pm \vec{b} = \langle a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \rangle.$$

Geometrically:

- 

- $\vec a - \vec b = \vec a + (-\vec b)$



Mnemonic rule:

$\boxed{\text{points from end of } \vec b \text{ to end of } \vec a}$

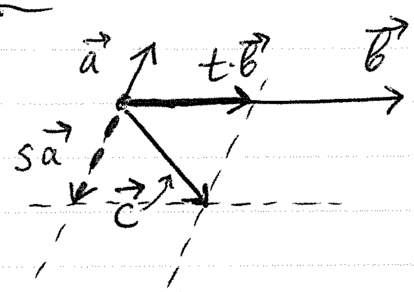
C 3 vectors in same plane

Suppose $\vec a$, $\vec b$, $\vec c$ lie in same plane, and $\vec a \nparallel \vec b$. Then:

$$\boxed{\vec c = s \cdot \vec a + t \cdot \vec b}$$

for some scalars s, t.

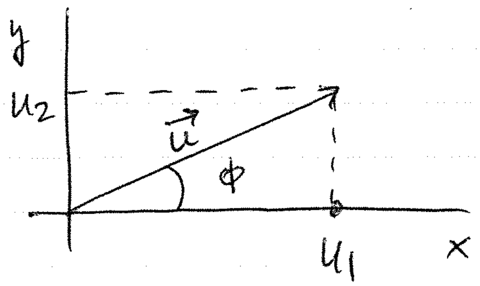
Geometric proof:



This will work for any $\vec c$.

$$\begin{pmatrix} s \cdot \vec a \parallel \vec a \\ t \cdot \vec b \parallel \vec b \end{pmatrix}$$

d Length & angle in 2D



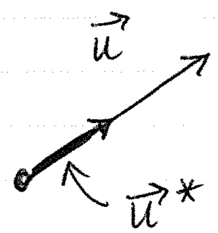
$|\vec{u}|$ = length of $\vec{u}$

$$\vec{u} = \langle u_1, u_2 \rangle$$
$$= \langle |\vec{u}|\cos\phi, |\vec{u}|\sin\phi \rangle.$$

Note: Must consistently use vector notations if you mean vectors:

$\vec{u} \leftarrow$ vector; $\quad u \leftarrow$ scalar.

e Unit vectors



$$\vec{u}^* = \frac{\vec{u}}{|\vec{u}|}$$

$\uparrow$ unit vector along $\vec{u}$.

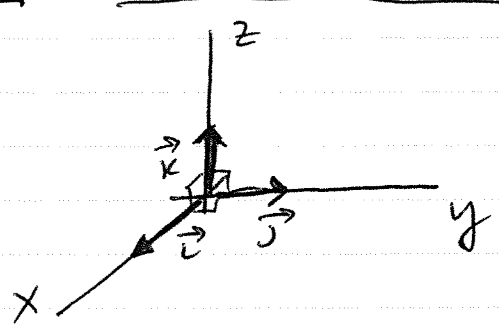
$|\vec{u}| \leftarrow$ length of $\vec{u}$

For example, a vector of length 3 and along $\vec{u}$ is:

$$\boxed{3 \cdot \vec{u}^*} \equiv \frac{3}{|\vec{u}|} \cdot \vec{u}.$$

length $\quad$ direction

f Unit coordinate vectors



$$\vec{i} \perp \vec{j} \perp \vec{k} \perp \vec{i}.$$

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1.$$

~~HW 1 (Sec. 12.1)~~

1-5

~~3, 12, $\underbrace{15, 23}_{\text{sphere}}$, $\underbrace{25, 29}$.~~

~~dist.~~

~~meaning of coords & distance~~

~~eq. of plane // coord. planes~~

~~HW2 (Sec. 12.2)~~

~~3, 9, 13, 43, 15 - basics, sum~~

~~25, 26 - unit vect; length/direction~~

~~31 - length & angle~~

~~45 - $\vec{c} = s\vec{a} + t\vec{b}$~~

~~47 - $|r - r_0| = 1$.~~