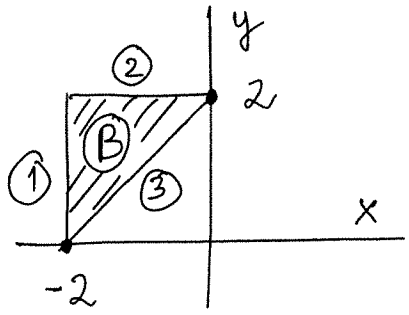
Ex. 4 This Example illustrates Note 1 on p 17-14: The corner points can be accounted for at the end. It also emphasizes the accurate way of finding the extrema **on the boundary sections of B**, which I already showed in Ex. 3. Note that similar examples on WebAssign are rushed and omit important details of the solution which you need to demonstrate on a quiz and/or test.



→ Find absolute extrema of $f(x,y) = xy^2 + x$ on the region B shown in the figure above.

Sol'n: **Step 1 of Algorithm**

Find cr. pts. inside B: $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} y^2 + 1 = 0 \\ 2xy = 0 \end{cases}$

The 1st equation can never hold true, therefore the system does not have a sol'n. **No cr. pts. inside B.**

Step 2 of Algorithm

On ① $x = -2$; $f(-2,y) = -2y^2 - 2 \equiv g(y)$ (call it)

Find cr. pts. on ①: $g'(y) = -4y = 0 \Rightarrow y = 0$.

Cr. pt. on ①: (-2, 0)

On ② $y = 2$: $f(x,2) = x \cdot 2^2 + x = 5x \equiv q(x)$

To find cr. pts. on ②: $g'(x) = 5 = 0$ ← never

No cr. pts. on ②

On ③: $y = x+2 \Rightarrow f(x, x+2) = x(x+2)^2 + x \overset{\text{algebra}}{=} \underbrace{x^3 + 4x^2 + 5x}_{g(x)}$

Find cr. pts. on ③: $g'(x) = 3x^2 + 8x + 5 = 0$

⇒ (by quadratic formula or by inspection ← just make sure it's right)

$(3x+5)(x+1) = 0 \Rightarrow x = -1$ and $x = -5/3$.

$y = -1+2 = 1$ $\qquad$ $y = -5/3 + 2 = 1/3$

Cr. pts on ③: $(-1, 1)$, $(-5/3, 1/3)$

Step 3 of Algorithm

The candidates for abs. extrema are:

Cr. pts. inside B :	none
Cr. pts. on ①, ②, ③ :	$(-2,0)$, $(-1,1)$, $(-5/3, 1/3)$
Corners :	$(-2,0)$, $(-2,2)$, $(0,2)$

$(-2,0)$ ← double counted

$f(-2,0) = -2 \cdot 0 - 2 = -2$

$f(-1,1) = -1 \cdot 1^2 - 1 = -2$

$f(-5/3, 1/3) = -5/3 \cdot 1/9 - 5/3 = -50/27$

$f(-2,2) = -2 \cdot 2^2 - 2 = \boxed{-10}$ ← abs. min @ $(-2,2)$

$f(0,2) = 0 \cdot 2^2 + 0 = \boxed{0}$ ← abs. max @ $(0,2)$

Answer.