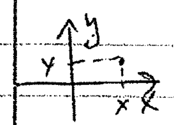
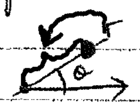
Sec. 15.7. Triple integrals in cyl. coordinates

1 Cylindrical coordinates

2D

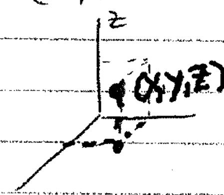
(x,y) & (r,θ)
cartesian polar





3D

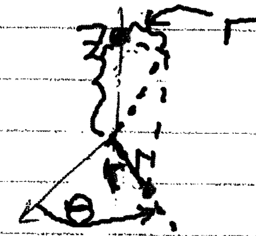
(x,y,z)



cartesian

polar "usual z"

(r,θ,z)

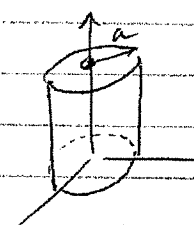


cylindrical.

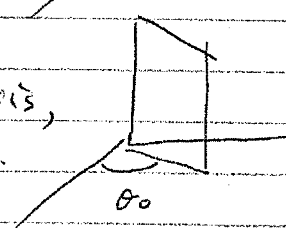
2 Surfaces in cyl. coord's

Cartesian: $x = \text{const}$ etc. $\to$ planes

Cylindrical: • $r = \text{const} = a$
cylinder of radius a,
axis $= z$-axis

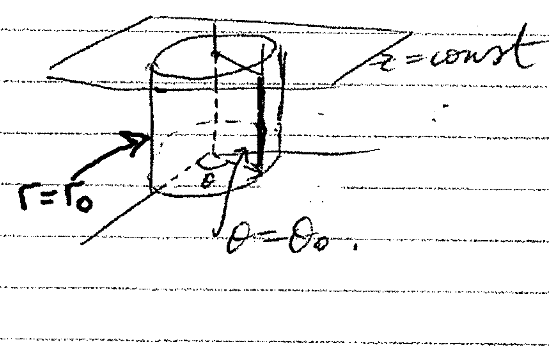


• $\theta = \text{const} = \theta_0$
half-plane containing z-axis,
at angle θ_0 to x-axis.



• $z = \text{const}$ – horizontal plane

Together:



Ex. 1 Write the surface in cart. coordinates and name it:

Will be used later often

(a) $z = r^2 \Rightarrow z = x^2 + y^2$ ← paraboloid

(b) $z = r \Rightarrow z = \sqrt{x^2+y^2}$ ← top part of cone

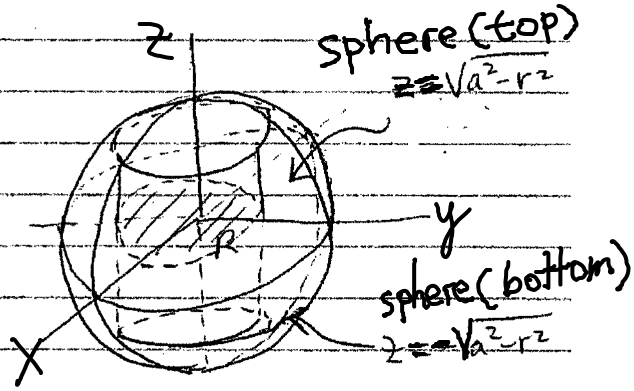
(c) $z = r(\cos\theta - \sin\theta) = r\cos\theta - r\sin\theta = x - y$

$z = x - y$ ← plane.

Ex. 2 The solid is the part of sphere $x^2+y^2+z^2=a^2$ that is also inside the cylinder $x^2+y^2=b^2$ $(b<a)$. Set up its volume as a triple int'l in cylindrical coordinates & cart. coord's.

Sol'n:

1) Sketch the solid



2) Rewrite the eqs. in cylindrical coord's:

Sphere: $r^2+z^2=a^2$, or $z^2=a^2-r^2$.

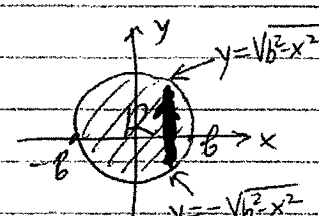
Cylinder: $r = b$

3) Set up the $\iiint$ in cylindrical coord's:

$$V = \iint_R \int_{z_{bot}}^{z_{top}} dz\,(r\,dr\,d\theta) = \int_0^{2\pi}\int_0^{b}\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} dz\; r\,dr\,d\theta.$$

4) Same volume in cartesian coord's:

$$V = \int_{-b}^{b}\int_{-\sqrt{b^2-x^2}}^{\sqrt{b^2-x^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dz\; dy\; dx.$$



The setup in cylindrical coord's looks much simpler. This is because we used the "natural" coord's for this solid.