

Calculus III

(1-1)

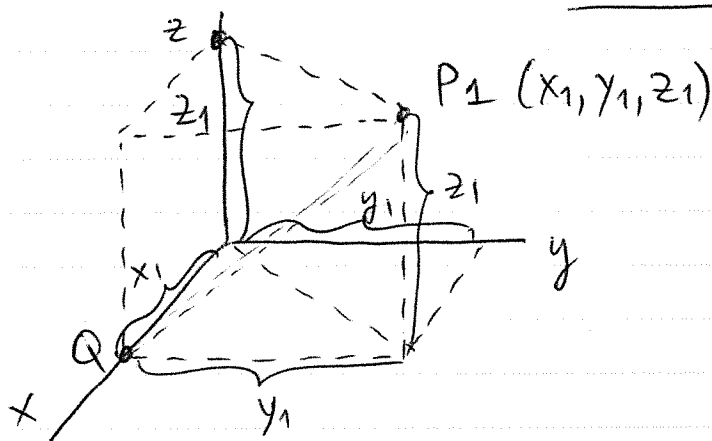
J. Stewart

Early Transcendentals

Lecture 1: Review of coordinates & vectors

① Coordinates (Sec. 12.1)

[a] Definition, distance to coord. planes and coord. axes



distance from P_1 to xy -plane:
 z_1 ;

dist. from P_1 to x -axis:

$$|P_1 Q| = \sqrt{y_1^2 + z_1^2}.$$

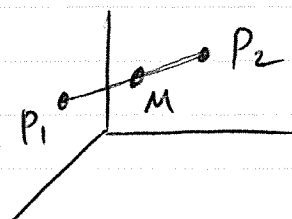
etc.

[b] Planes $\parallel$ to coord. planes

$x = x_1 \leftarrow$ plane $\parallel$ zy -plane,
 x_1 units away from it
(contains Q, P_1 in the figure).

[c] Midpoint of segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).$$



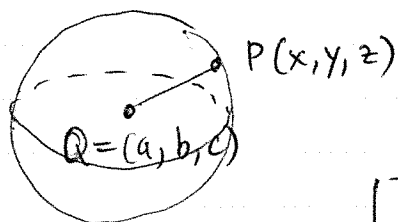
[d] Distance between points :

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$$|P_1 P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(like the Pythagorean Thm. in 3D).

[e] Sphere : all points equidistant from center $Q(a, b, c)$:

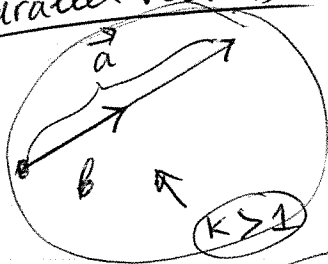


$$|PQ| = R, \text{ or}$$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = R$$

(2) Vectors (Sec. 12.2)

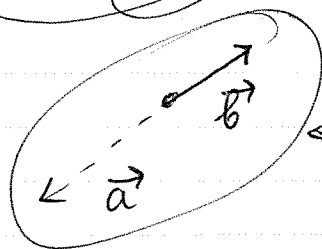
[a] Parallel vectors



$$(\vec{a} \parallel \vec{b}) \Leftrightarrow (\vec{a} = k \cdot \vec{b})$$

for some scalar k .

$$k = \frac{\text{length } \vec{a}}{\text{length } \vec{b}} = \frac{|\vec{a}|}{|\vec{b}|}$$



$$\leftarrow k < 0.$$

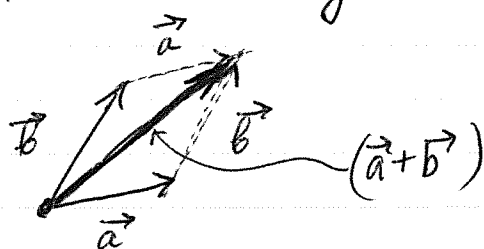
[b] Sum & difference of vectors

Algebraically:

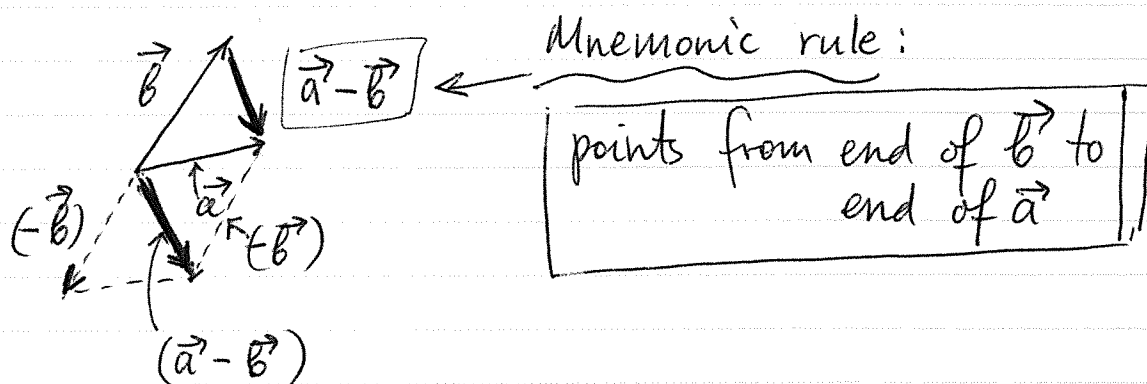
$$\vec{a} \pm \vec{b} = \langle a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \rangle.$$

Geometrically:

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• $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$



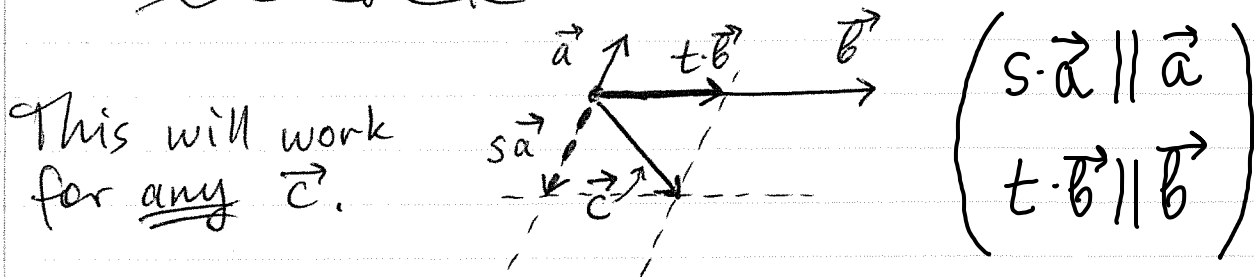
[c] 3 vectors in same plane

Suppose $\vec{a}$, $\vec{b}$, $\vec{c}$ lie in same plane,
and $\vec{a} \nparallel \vec{b}$. Then:

$$\vec{c} = s \cdot \vec{a} + t \cdot \vec{b}$$

for some scalars s, t .

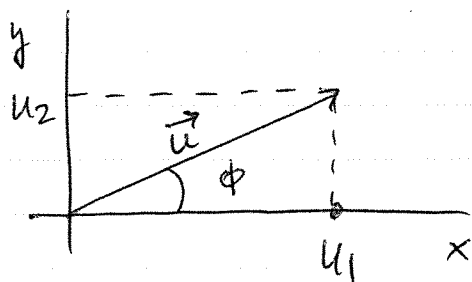
Geometric proof:



[d]

Length & angle in 2D

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$$|\vec{u}| = \text{length of } \vec{u}$$

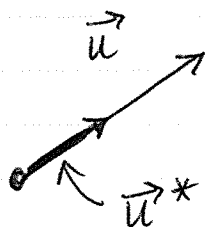
$$\vec{u} = \langle u_1, u_2 \rangle$$

$$= \langle |\vec{u}| \cos \phi, |\vec{u}| \sin \phi \rangle$$

Note: Must consistently use vector notations if you mean vectors:
 $\vec{u} \leftarrow \text{vector}; \quad u \leftarrow \text{scalar}.$

[e]

Unit vectors



$$\vec{u}^* = \frac{\vec{u}}{|\vec{u}|} \leftarrow \begin{array}{l} \text{unit} \\ \text{vector} \\ \text{along } \vec{u}. \end{array} \quad \begin{array}{l} \text{length} \\ \text{of } \vec{u} \end{array}$$

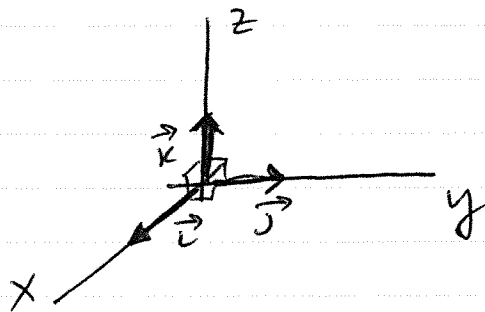
For example, a vector of length 3 and along $\vec{u}$ is:

$$\boxed{3 \cdot \vec{u}^*} \equiv \frac{3}{|\vec{u}|} \cdot \vec{u}.$$

length direction

[f]

Unit coordinate vectors



$$\vec{i} \perp \vec{j} \perp \vec{k} \perp \vec{i}.$$

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1.$$