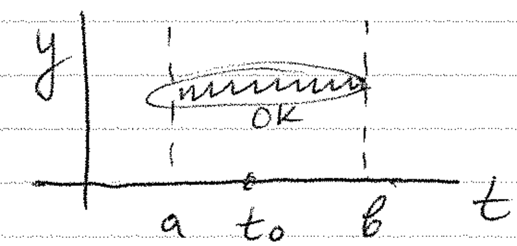
Lecture 6 Existence & uniqueness of solution of nonlinear DEs

Recall Thm. 2.1 for linear DEs:

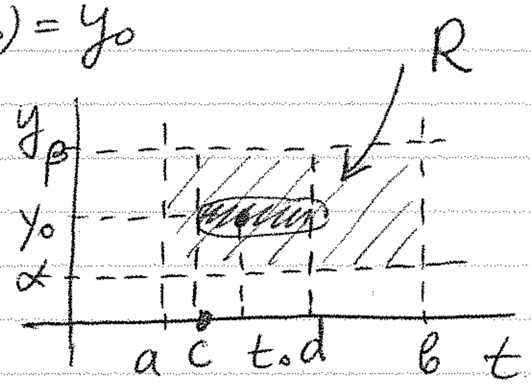
$$y' + p(t)y = g(t)$$
$$y(t_0) = y_0$$



$$\begin{pmatrix} p(t), g(t) \\ \text{continuous} \\ \text{on } (a,b) \end{pmatrix} \Rightarrow \begin{pmatrix} y(t) \\ \text{exists \&} \\ \text{is unique on } (a,b) \end{pmatrix}$$

Thm. 2.2 (for nonlinear DEs)

$$y' = f(t,y), \quad y(t_0) = y_0$$



- $f(t,y)$ & $\frac{\partial f(t,y)}{\partial y}$ are functions of (t,y) $\Rightarrow$ they are defined in the (t,y)-plane.

- Suppose f & f_y are continuous in some $R = (a,b) \times (\alpha, \beta)$ in (t,y)-plane, and let $(t_0, y_0) \in R$.

Then $y(t)$ is guaranteed to exist for *some* $t \in (c,d)$, where (c,d) is *inside* (a,b). However, we cannot predict how wide (c,d) is.

<u>Note:</u> This should be contrasted with the situation for linear DEs: There, if $f(t,y) = -p(t)y + g(t)$ and $f_y = -p(t)$ are continuous on $t \in (a,b)$, then the solution is guaranteed to exist on the <u>entire</u> (a,b).

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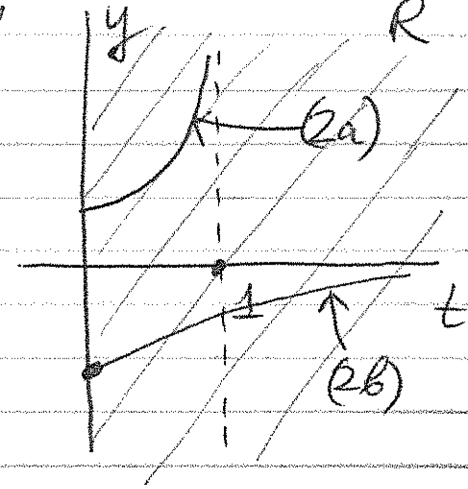
An example where (c,d) may or may not be the entire (a,b) for a <u>nonlinear</u> DE was illustrated by $\overline{\text{Ex. } 2(a,b)}$ of Lecture 5.

<u>Recall</u>:

<u>Ex. 2</u> $y' = y^2 \Rightarrow f(t,y) = y^2$;
$f_y(t,y) = 2y$
are continuous on
$(t,y) \in (0,\infty) \times (-\infty,\infty)$



(2a) $\underline{y(0) = 1} \Rightarrow y = \frac{1}{1-t}$
$\Rightarrow$ blows up @ $t = 1$
$\Rightarrow (c,d) = (0,1)$ while
$(a,b) = (0,\infty)$.

2(b) $\underline{y(0) = -1} \Rightarrow y = \frac{-1}{1+t} \Rightarrow$ exists for $t \in (0,\infty)$
$\Rightarrow (c,d) = (0,\infty) \leftarrow$ same as (a,b).

<u>So, to repeat:</u>

All that the thm, for nonlinear eqs, is able to say is that the solution of the IVP exists only <u>sufficiently near</u> the initial point $t=t_0$.