9. Problem $|T(y) - \tilde{T}(y)| \le 400\epsilon\sqrt{y}$.

12. Problem $T_n(y) \to y/2$, and the shape corresponding to this drain time function is $f(y) = y^{1/4}/(10\sqrt{2})$. However,

$$T_n'(1/2) = 1/2 + 3\sqrt{n}/2 \to \infty.$$

Therefore, $f_n(1/2) \to \infty$.

15. Problem Since

$$dV/dt = -a\sqrt{2gy},$$

if dV/dy were a constant, then y would be constant; i.e., no water would drain from the tank. For the cylinder in question,

$$\sqrt{y} = -\frac{2 \times 10^{-3}}{\pi}t + \sqrt{.4}.$$

For $t \in [0, 3]$ the values of dV/dt differ from $dV/dt(1.5)$ by less than 2 percent.

A.2.3 What Goes Around Comes Around

1. Exercise $\ddot{\mathbf{r}} = -a^2\mathbf{r}$, so the force is central. Since $\|\dot{\mathbf{r}}\| = ar$, we have

$$\|\ddot{\mathbf{r}}\| = \|\dot{\mathbf{r}}\|^2/r.$$

2. Exercise Direct calculation.

3. Problem

$$\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \frac{(\mathbf{r}\cdot\mathbf{r})\dot{\mathbf{r}} - (\dot{\mathbf{r}}\cdot\mathbf{r})\mathbf{r}}{r^3} = \frac{(\mathbf{r}\times\dot{\mathbf{r}})\times\mathbf{r}}{r^3}$$

by Exercise 2.

5. Question No. Just about any example works, e.g., $\mathbf{r} = t\mathbf{i} + e^t\mathbf{j}$.

6. Problem Draw a line through the center of force parallel to the line of motion. Consider the areas of the triangles swept out by the radial segment over two consecutive time intervals of the same duration.

7. Exercise The second derivative of a linear function vanishes.

8. Exercise Differentiate $\frac{1}{2}r^2\dot{\theta} = \text{constant}$.

9. Problem If the force is central, then it is planar and hence may be parameterized in the form

$$\dot{\mathbf{r}} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}.$$

Compute $\ddot{\mathbf{r}}$ and use the result of Exercise 8.

10. Problem If the orbit is represented by $x = r\cos\theta$, $y = r\sin\theta$, then

$$x\dot{y} - y\dot{x} = r^2\dot{\theta} = \text{constant}.$$

Therefore, $x\ddot{y} - y\ddot{x} = 0$, and hence

$$\mathbf{r}\times\ddot{\mathbf{r}} = (x\mathbf{i} + y\mathbf{j})\times(\ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}) = \mathbf{0};$$

i.e., the force is centrally directed.

11. Problem $r\dot{\theta}^2 = r(hu^2)^2 = h^2u^3$.

12. Problem Since $r = u^{-1}$, Problem 11 gives

$$\dot{r} = -u^{-2}\frac{du}{d\theta}\dot{\theta} = -u^{-2}\frac{du}{d\theta}(hu^2) = -h\frac{du}{d\theta}.$$

Now differentiate again.

14. Problem Here, $u = 1/r = A\csc\theta$ where $A = 1/a$. This leads to

$$\frac{d^2u}{d\theta^2} + u = 2A\csc^3\theta \propto u^3,$$

and hence the force is proportional to $u^2u^3 = r^{-5}$.

15. Problem If $r = a/(1 + e\cos\theta)$, then $u = A + B\cos\theta$ and hence $(d^2u/d\theta^2) + u$ is a constant.

Therefore an inverse-square force acts.

16. Problem Inverse cube.

17. Problem The force has the form $A/r^5 + B/r^3$.

18. Problem Here, $abu = (b^2\cos^2\theta + a^2\sin^2\theta)^{1/2}$, where a and b are the semiaxes of the ellipse. Routine, but tedious, calculations give $(d^2u/d\theta^2) + u \propto 1/u^3$. So the force is proportional to r.

19. Problem The force is *repulsive* and proportional to r.

20. Problem The force is proportional to a linear combination of $1/r^3$ and $1/r^7$.