MATH 235 — Mathematical Models and Their Analysis

Instructions: Present your work in a neat and organized manner. Please **use** either the 8.5×11 size paper or the filler paper with pre-punched holes. Please do **not use** paper which has been torn from a spiral notebook. Please secure all your papers by using either a staple or a paper clip, but **not** by folding its (upper left) corner.

You **must** show **all of the essential details** of your work to get full credit. If you used *Mathematica* for some of your calculations, attach a printout showing your commands and their output. If I am forced to fill in gaps in your solution by using notrivial (at my discretion) steps, I will also be forced to reduce your score.

Please refer to the syllabus for the instructions on working on homework assignments with other students and on submitting **your own** work.

Homework Assignment # 1

1. (a) Show that for the secondary rainbow, the angle by which a ray is turned by the raindrop is

$$T(\alpha) = 360^{\circ} + 2\alpha - 6\beta,$$

where α and β are the angles introduced in Lecture 1 for the primary rainbow.
You must present a clear explanation with a clearly labeled picture, simular to that found at the end of p. 3.

(b) Show that for the "secondary rainbow ray" (defined similarly to the "primary rainbow ray"),

$$\cos^2 \alpha_0 = \frac{n^2 - 1}{8},$$

and find the expression for the corresponding $T(\alpha_0)$. Use $n = 1.33$ to find the numeric value for $T(\alpha_0)$ in degrees. Sketch a picture that clearly explains at what angle relative to the sun, and why, the observer sees a secondary rainbow. (For an example of such a picture for the primary rainbow, see the last two figures in Sec. 1.2.)

2. (a) What is the angular width of the secondary rainbow? (Use the mthematically literate approach.)

(b) Which color is at the top in the secondary rainbow? You must present a *clear* explanation along the lines of Sec. 1.3.

Bonus Credit (**0.5 point**) will be given only if the solution is mostly correct and *clearly explained*. In particular, you must draw a picture (or pictures) accompanying your explanation.

Plot the $T(\alpha)$ from Problem 1(a) versus α (e.g., use *Mathematica*). You will see that the α_0 defined in Problem 1(b) corresponds to a minimum of $T(\alpha)$. This, however, contradicts the picture of Alexander's dark band in Sec. 1.6. Alexander's dark band is a real physical phenomenon, and hence somehow your conclusion that $T(\alpha)$ has a minimum at α_0 for the secondary rainbow, must be incorrect. It must have a maximum, as the figure in Sec. 1.6 shows.

Resolve this seeming paradox.

Note: Disregard the color in this problem; consider only the "average" ray, as in Sec. 1.2.

Hint: Draw a raindrop and show the range of all outgoing rays that can form the primary rainbow. In doing so, use the information about $T(\alpha)$ from Sec. 1.2. Now, in the same picture, repeat this for the secondary rainbow, where now you need to use the information from the plot of $T(\alpha)$ which you computed in Problem 1(a).