MATH 235 — Mathematical Models in Science and Engineering

Instructions: Present your work in a neat and organized manner. Please **use** either the 8.5×11 size paper or the filler paper with pre-punched holes. Please do **not use** paper which has been torn from a spiral notebook. Please secure all your papers by using either a staple or a paper clip, but **not** by folding its (upper left) corner.

You **must** show **all of the essential details** of your work to get full credit. If you used *Mathematica* for some of your calculations, attach a printout showing your commands and their output. If I am forced to fill in gaps in your solution by using notrivial (at my discretion) steps, I will also be forced to reduce your score.

Please refer to the syllabus for the instructions on working on homework assignments with other students and on submitting **your own** work.

Homework Assignment # 13

1. Parts (a) and (b) are not related to each other and *each* is worth **(0.5 point)**.

   (a) Verify that Eq. (35) in the Notes is equivalent to Eq. (6).

   (b) Follow the lines of the derivation of Eq. (34) to derive Eq. (43).

2. The convergent improper integral

$$\int_0^\infty \frac{s^m}{e^s - 1}\, ds \qquad \text{with} \quad m = 3 \tag{HW13.1}$$

   played a prominent role in the development of Quantum Mechanics: see Eq. (38) in the Notes. The value of this integral can be found using the method described below. You will have the opportunity to find that value in the Bonus Problem. Here, however, you will need to use that method to show that the above integral with $m = 0$ diverges. Follow these steps.
   (i) Expand the integrand above (with $m = 0$) into a *converging* geometric series. Use the convergence test for the geometric series to explain why the series that you have obtained is converging.
   Hint: In this series, $r \neq s$. An insight as to what r *is* can be found in one of the derivations of Sec. 13.4.
   (ii) Integrate each term of this series; this will give you a new series. This should be a p-series, which you studied in Calculus II.
   (iii) Explain why this new series diverges. This will imply that the original integral diverges.
   (iv) Finally, follow an analysis found in Sec. 13.5 to identify which of the integration limits this divergence is coming from. In doing so, you will need to reference a certain type of indefinite integral.

   Food for thought Remembering where the variable s came from, which "catastrophe" does this divergence manifest: ultraviolet or infrared (see Sec. 13.5)?

3. This problem is worth **0.5 point**.

   The main purpose of this problem is to make you familiar with the wavelengths (and perhaps frequencies) of principal colors. In the process you will also have to get acquainted with values of the Planck and Boltzmann constants.

   Download the code `hw13_p3.m` from the Homework website. Follow the instructions inside the code: get the parameter values and complete some minor programming, as indicated there. Make sure to use the dimensional units specified in the code.

Print both plots and attach them to your work. Discuss whether one of the plots is in better correspondence to the "accepted" colors of the three stars than the other plot. How good is this correspondence for this "better" plot?

Bonus (worth **0.5 point**; credit will be given only if the solution is mostly correct)
Follow the approach outlined in Problem 2 to find the value of the integral (HW13.1) with $m = 3$.
At step (ii) in that approach, you may need to make a change of variables so that each term of the series that you are integrating would contain the same factor e^{-y}.
After completing step (ii), you will obtain a converging p-series. Another name for a p-series is *Riemann zeta function*. Search for this name online, or ask Mathematica, and find the value of the specific zeta function that you need.