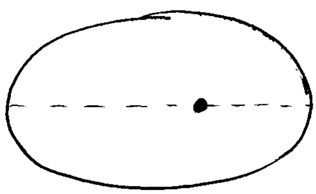
Sol. 2-1

Problem 4, HW 2

1) $$A = \frac{1}{2}\int_0^{2\pi}\left(\frac{c^2/N}{1+e\cos\theta}\right)^2 d\theta = \frac{\pi c^4/N^2}{(1-e^2)^{3/2}}$$

2) $$T = \frac{A}{\dot{A}} = \frac{\pi c^4/N^2}{(1-e^2)^{3/2}} \cdot \frac{2}{c} = \frac{2\pi c^3/N^2}{(1-e^2)^{3/2}}.$$

3)



$$\begin{aligned} 2a &= r(0) + r(\pi) \\ &= eD\left[\frac{1}{1+e} + \frac{1}{1-e}\right] \\ &= \frac{2eD}{1-e^2} = \frac{2c^2/N}{1-e^2} \end{aligned}$$

4) $$\frac{T^2}{a^3} = \frac{4\pi^2 c^6/N^4}{(1-e^2)^3} : \frac{c^6/N^3}{(1-e^2)^3}$$

$$= \frac{4\pi^6}{N}$$ ← independent of e and c (which are specific to the planet).

Sol. 2-2

Problem 5, HW 2

1) $(\vec{r} \times \ddot{\vec{r}}) \times \vec{r} = \vec{0}$

$$(\vec{r}\cdot\vec{r})\ddot{\vec{r}} - (\ddot{\vec{r}}\cdot\vec{r})\vec{r} = \vec{0}$$

$$r^2\ddot{\vec{r}} = (\ddot{\vec{r}}\cdot\vec{r})\vec{r}$$

$$r^2\ddot{\vec{r}} = \left((\dot{\vec{r}}\cdot\vec{r})^{\cdot} - (\dot{\vec{r}}\cdot\dot{\vec{r}})\right)\vec{r}$$

The 1st term on the rhs:

$$(\dot{\vec{r}}\cdot\vec{r})^{\cdot} = (r\dot{r})^{\cdot} = \dot{r}^2 + r\ddot{r}$$

The 2nd term on the rhs:

$$\dot{\vec{r}}\cdot\dot{\vec{r}} = \dot{x}^2 + \dot{y}^2 = \dot{r}^2\cos^2\theta - \cancel{2\dot{r}\dot{\theta}\cos\theta\, r\sin\theta} + \dot{\theta}^2 r^2\sin^2\theta$$
$$+ \dot{r}^2\sin^2\theta + \cancel{2\dot{r}\dot{\theta}\sin\theta\, r\cos\theta} + \dot{\theta}^2 r^2\cos^2\theta$$
$$= \dot{r}^2 + \dot{\theta}^2 r^2.$$

Combining the 2 terms:

$$r^2\ddot{\vec{r}} = (\cancel{\dot{r}^2} + r\ddot{r} - \cancel{\dot{r}^2} - \dot{\theta}^2 r^2)\vec{r}$$

$$r^2\ddot{\vec{r}} = (r\ddot{r} - \dot{\theta}^2 r^2)\vec{r}.$$

Problem 5, HW2 (cont'd)

2) $$u = \frac{1}{r}, \qquad c = h$$

$$\dot{r} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dr}{d\theta}\cdot\frac{c}{r^2}$$

$$= -c\,\frac{d}{d\theta}\left(\frac{1}{r}\right) = -c\,\frac{du}{d\theta}.$$

$$\ddot{r} = -c\cdot\frac{d}{dt}\left(\frac{du}{d\theta}\right) = -c\,\frac{d}{d\theta}\left(\frac{du}{d\theta}\right)\cdot\frac{d\theta}{dt}$$

$$= -c\cdot\frac{d^2u}{d\theta^2}\cdot\frac{c}{r^2} = -c^2u^2\cdot\frac{d^2u}{d\theta^2}.$$

3) $$r^2\ddot{\vec{r}} = (r\ddot{r} - \dot{\theta}^2\cdot r^2)\,\vec{r}$$

$$r\cdot\frac{1}{u}\cdot\ddot{\vec{r}} = \left(\frac{1}{u}\cdot\left[-c^2u^2\frac{d^2u}{d\theta^2}\right] - \left(\frac{c}{r^2}\right)^2\cdot\frac{1}{u^2}\right)\vec{r}$$

$$\ddot{\vec{r}} = \frac{\vec{r}}{r}\left(-c^2u^2\,\frac{d^2u}{d\theta^2} - c^2\cdot u^4\cdot\frac{1}{u}\right)$$

$$\ddot{\vec{r}} = \frac{\vec{r}}{r}\left(-c^2u^2\left[\frac{d^2u}{d\theta^2} + u\right]\right).$$

Sol. 2-4

Problem 6, HW 2

Will do # 17; the other 3 problems are no more difficult.

$$r = a\theta \,, \qquad \frac{1}{r} = \frac{1}{a}\frac{1}{\theta} \iff u = \frac{1}{a}\cdot\frac{1}{\theta}.$$

$$-c^2u^2\left[\frac{d^2u}{d\theta^2} + u\right] = -c^2u^2\left[\frac{2}{a}\frac{1}{\theta^3} + \frac{1}{a}\frac{1}{\theta}\right]$$

$$= -\frac{c^2}{r^2}\left[\frac{2a^2}{(a\theta)^3} + \frac{1}{a\theta}\right]$$

$$= -\frac{c^2}{r^2}\left[\frac{2a^2}{r^3} + \frac{1}{r}\right]$$

$$= -c^2\left[\frac{2a^2}{r^5} + \frac{1}{r^3}\right].$$