LXXI. *On Induced Stability*. By ANDREW STEPHENSON *.

1. NOT only may a single pivoted body be maintained in the position of unstable equilibrium by vertical oscillation of the pivot, but furthermore a number of freely jointed links may be similarly maintained.

In the simplest case, when there are only two links, the conditions of stability are readily found. With the usual notation the equations of motion in the neighbourhood of the vertical are

$$\{(h_1^2+k_1^2+\mu l_1^2)D^2-(g+2\alpha n^2\cos nt)(h_1+\mu l_1)\}\theta+\mu l_1 h_2 D^2\phi=0,$$
$$\{(h_2^2+k_2^2)D^2-(g+2\alpha n^2\cos nt)h_2\}\phi+l_1 h_2 D^2\theta=0,$$

where μ is the ratio of the mass of the second, or upper, rod

* An addition to a paper under the above title, Feb. 1908. Communicated by the Author.

to that of the first, or lower, and α is half the amplitude of the applied motion. Writing these equations

$$[a\mathrm{D}^2-\{g+\alpha n^2(e^{int}+e^{-int})\}]\theta+b\mathrm{D}^2\phi=0,$$
$$[p\mathrm{D}^2-\{g+\alpha n^2(e^{int}+e^{-int})\}]\phi+q\mathrm{D}^2\theta=0,$$

we have two particular solutions, each of the form

$$\theta=\sum_{-\infty}^{\infty}\mathrm{A}_r e^{(c+rin)t},\quad \phi=\sum_{-\infty}^{\infty}\mathrm{B}_r e^{(c+rin)t},$$

where

$$\left.\begin{array}{l}\{-g+a(c+rin)^2\}\mathrm{A}_r-\alpha n^2(\mathrm{A}_{r-1}+\mathrm{A}_{r+1})+b(c+rin)^2\mathrm{B}_r=0\\ \{-g+p(c+rin)^2\}\mathrm{B}_r-\alpha n^2(\mathrm{B}_{r-1}+\mathrm{B}_{r+1})+q(c+rin)^2\mathrm{A}_r=0\end{array}\right\}. \quad (r)$$

The set of conditional equations, (r), determines c and the relative values of the coefficients. When α is small the terms diminish rapidly from A_0 and B_0, and when α approaches the limit zero, αn remaining finite, we obtain

$$(ap-bq)^2c^4+\{2(\alpha n)^2(a^2+p^2+2bq)-g(a+p)(ap-bq)\}c^2$$
$$+4(\alpha n)^4-2(\alpha n)^2g(a+p)+g^2(ap-bq)=0.$$

The roots of this quadratic in c^2 are real for all values of αn. For stability the quantities

$$2(\alpha n)^2(a^2+p^2+2bq)-g(a+p)(ap-bq)$$

and

$$4(\alpha n)^4-2(\alpha n)^2g(a+p)+g^2(ap-bq)$$

must be positive. Thus stability is always ensured by making the frequency of the applied motion sufficiently large. For two equal rods, each of length l, the condition is

$$(\alpha n)^2>0{\cdot}683lg.$$

It may be noted for the sake of comparison that for a single rod of length $2l$, for stability $(\alpha n)^2>\frac{2}{3}lg$.

2. In the case of a chain of three uniform rods, each of length l, we obtain the c equation by a method similar to the preceding:—

$$26(cl)^6+9(2594\mu-41lg)(cl)^4+9\{154156\mu^2-11572\mu lg+112(lg)^2\}(cl)^2$$
$$+81\{27040\mu^3-12480\mu^2lg+570\mu(lg)^2-5(lg)^3\}=0,$$

where $\mu=\frac{3}{13}(\alpha n)^2$, and α is small.

The roots of this equation in c^2 are real for all values of αn. They are negative if

$$(\alpha n)^2>1{\cdot}79lg,$$

the condition for stability.

March, 1909.