Descartes and the Radius of the Rainbow

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Descartes and the Radius of the Rainbow

BY CARL B. BOYER *

FEW scientists have been subjected to charges of plagiarism more frequently than Descartes (1596–1650). In particular, each of the appendices to the *Discours de la méthode* has been questioned. *La dioptrique* long was believed to have been derived from work of Snell;[1] *Les météores* once was regarded as inspired by the ideas of Kepler and De Dominis;[2] and in *La géométrie* some critics thought they saw unacknowledged indebtedness to Harriot and Oresme.[3] None of these charges has been substantiated; yet in each case Descartes was neither so original nor so definitive as he had presumed. In each of the appendices he had been anticipated by some, corrected by others, on at least one essential point. Thus, as is now well known, the Cartesian theory of the rainbow had been given in qualitative form by Theodoric of Freiburg (as well as by Muslim scholars) in the fourteenth century,[4] and in the eighteenth century it was superseded by the theory of Young and Airy.[5] It is the purpose of this note to call attention to an overlooked detail in his work on the rainbow in which Descartes failed adequately to appreciate his relationship to his predecessors and successors.

Descartes believed, mistakenly, that he was the first one to study the rainbow through experiments with a large spherical globe of water which served as a magnified raindrop. On the basis of observations and calculations of the paths of a great number of rays (Descartes reports that he studied 10,000!) striking the upper half of the globe, he found that the radius of the rainbow is about 42°.[6] More accurately, he placed the greatest radius at 41°47′ and the smallest at about 40°. Descartes was justifiably proud of this achievement, but he carried his boast too far. He claims to have been aware of but one earlier estimate of the size of the bow, a value of 45° given "par la créance commune" and which he ascribed to Maurolycus.[7] On the basis of this impression Descartes remarked, "Ce qui monstre le peu de foy qu'on doit adiouster aux observations qui ne sont pas accompagnées de la vraye raison." [8] Yet Maurolycus himself had queried, following his purported demonstration that the radius is 45°, "But how does it happen, you ask, that the altitude of the rainbow is not exactly 45°, but a little less as ascertained by observation? I do not know how to answer this or what reason I may offer, unless it be that the falling drops are somewhat elongated or somewhat flattened, and thus, varying from the spherical form, change the angle of reflection and hence also the straightness of the ray which in the case of a perfect sphere comes back at an angle of forty-five degrees." [9] Hence, either Maurolycus himself had made a more careful measurement or else he was aware of an older and more accurate determination. That the latter probably was the case is indicated by the fact that closer approximations were given by his contemporaries. In

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[1] P. Kramer, "Descartes und das Brechungsgesetz des Lichtes," *Abhandlungen zur Geschichte der Mathematik*, *4* (1882), 233–278.

[2] Leibniz, *Opera omnia*, *5* (Genevae, 1768), p. 547; and R. E. Ockenden, "Marco Antonio de Dominis and his explanation of the rainbow," *Isis*, *26* (1936), 40–49.

[3] *Oeuvres de Descartes* (edited by Charles Adam and Paul Tannery, 12 vols., Paris, 1897–1913), *12*, 215.

[4] Giambatista Venturi, *Commentarj sopra la storia e le teorie dell' ottica*, vol. I (only one published, Bologna, 1814), pp. 149–180.

[5] Friedrich Just, *Geschichte der Theorien des Regenbogens* (17 p., Marienburg, 1863).

[6] *Oeuvres*, *6*, 327, 336, 343, 700, 705, 708.

[7] Maurolycus' work on the rainbow appeared in the *Photismi de lumine*, written in 1554 and appearing in numerous editions from 1575 on. I have used an edition of this work with the title *Theoremata de lumine* (Lugduni, 1617), as well as the English translation by Henry Crew of the *Photismi de lumine* (New York, 1940).

[8] *Ouevres*, *6*, 340, 708.

[9] *Theoremata*, p. 90; *Photismi* (Crew), p. 93.

1540 Alessandro Piccolomini had appended a little tract, *De iride*, to his translation of the Commentary of Alexander Aphrodisias on the *Meteorologia* of Aristotle; and in this he had asserted that the rainbow can be as high as 42°. If the sun has an elevation of more than 42°, he continued, no rainbow is visible, for the bow can be seen only if the sun's elevation is less than 42°. This fact is based, he claimed, upon many observations.[10] Again, ten years later, Cardan wrote in his popular work *De subtilitate* that "in our regions the maximum elevation of the rainbow above the horizon can not be more than forty-two parts."[11] In 1571 a comparable figure is given by Johann Fleischer in his lengthy *De iridibus doctrina Aristotelis et Vitellionis certa methodo comprehensa.* Here the author accepts the principle that the altitude of the sun added to the altitude of the bow will give the radius of the rainbow, although he believed that the last quantity varied slightly with atmospheric conditions. Thus, when the sun is rising, he took a radius of 42° 30′ for the bow; but when the altitude of the sun is 28° 24′, he accepted an altitude of 13° 36′, giving a radius of just 42°.[12]

The best known work on the rainbow in the early seventeenth century was contained in the *De radiis visus et lucis* of DeDominis, published in 1611 but written, the author held, some twenty years earlier. Here one finds an explanation of the bow which comes close to that of Theodoric of Freiburg, although it contains some gross errors which lead one to question the author's true understanding;[13] and again one reads that no rainbow is seen if the sun is higher than 42°, according to certain observations.[14] Snell in Holland must have been familiar with this work, for in marginal notes to Risner's edition (1572) of the Optics of Alhazen and Witelo he wrote, "DeDominis makes the greatest height 42°."[15]

From whom were Descartes' predecessors taking the ubiquitous value of 42°? The chances are that most of them were taking it from the very work upon the margin of which Snell jotted his notation — the *Opticae* of Witelo. Although most of Witelo's work is a close commentary upon the *Opticae* of Alhazen, the sections on the rainbow with which the commentary closes apparently were independent; and here, in a book written between 1270 and 1278, one reads that "some have observed that the height of the bow and the sun are together always just 42°."[16] Witelo, however, contended that atmospheric refraction will make some small difference in the radius. Among the observers to whom Witelo refers one must include Roger Bacon. In the *Opus majus* of 1269 the author advises the experimenter to take "the required instrument and look through the openings of the instrument" to find that the higher the sun, the lower is the rainbow. He reports that the experimenter will find the maximum elevation of the rainbow to be 42°, a value repeated several times throughout the book.[17] Was this value original with him? If so, this would indicate that, Descartes notwithstanding, there seems to be little connection between precision of measurement and correctness of theory in pre-Cartesian works on the rainbow. Bacon's views on the bow were elementary in the extreme as compared to those of Theodoric early in the century following, yet the quantitative statements of the latter are thoroughly bad. For the primary bow Theodoric adopted the fantastically inadequate radius of 22°.[18] It is, of

[10] *Alexander Aphrodisiensis . . . in quatuor libros meteorologicorum Aristotelis . . . quam Latinitate donavit Alexander Piccolomineus . . . Accedit insuper eiusdem Alexandri Piccolominei tractatus de iride* (Venetiis, 1540), folio 63 verso, column 2.

[11] Cardan, *Opera* (10 vols., Lugduni, 1663), 423.

[12] *De iridibus doctrina*, pp. 173–174. The title-page bears the imprint, Witebergae, 1579, but the colophon is dated 1571.

[13] R. E. Ockenden, "Marco Antonio de Dominis and his explanation of the rainbow," *Isis*, *26* (1936), 40–49.

[14] *De radiis visus et lucis* (Venetiis, 1611), p. 66.

[15] Christiaan Huygens, *Oeuvres complètes* (22 vols., La Haye, 1888–1950), *17*, 357.

[16] Witelo, *Opticae* (ed. by Risner, Basileae, 1572), p. 471.

[17] *The opus majus of Roger Bacon* (transl. by R. B. Burke, 2 vols., Philadelphia, 1928), *2*, 592.

[18] Engelbert Krebs, "Meister Dietrich (Theodoricus Teutonicus de Vriberg). Sein Leben, seine Werke, seine Wissenschaft", *Beiträge zur Geschichte der Philosophie des Mittelalters*, *5* (1905–1906), Heft 6, p. 29. Cf. also *12* (1914), Heft 5–6.

course, possible that Theodoric copied this incorrectly from the figure given by Bacon and Witelo; but the fact remains that he used 22° throughout all of his calculations. It may be that numerological considerations led him to his value, for he placed the radius of the halo at 11°, that of the primary rainbow at 22°, and that of the secondary bow at 33°. This last measure is the earliest that I have found for the secondary, and while it is in itself a very poor approximation, nevertheless the difference between the radii of the primary and secondary bows is not bad. It is, in fact, almost the same as that given about a quarter of a millennium later by Maurolycus. The geometrically-minded Maurolycus, having taken the radius of the primary rainbow to be half of a right angle, believed that the secondary was larger by an eighth of a right angle — i. e., by 11°¼.[19] Descartes later measured the radius of the secondary bow as about 51° or 52°, or about 10° larger than that of the primary rainbow.[20]

The object of this paper has been principally to call attention to the many accurate measurements of the rainbow before the time of Descartes. The value 42° runs like a golden thread through at least half a dozen published works from 1269 to 1611. If Descartes was unaware of any of these anticipations, one can only conclude that he had a remarkable facility for overlooking, in the works of his predecessors, anything which might be of value in connection with his own discoveries.[21] In fairness to Descartes, however, it must immediately be pointed out that if he was not first to measure the radius of the bow accurately, he nevertheless was the first one to give a reasonably satisfactory theoretical justification for this radius. Huygens regarded this as "about the only part of Cartesian physics which was well taken." [22]

Descartes all too frequently exaggerated the finality of his explanations, and his radius of the rainbow was no exception. *Les météores* closes with the characteristic hope that "those who have understood all which has been said in this treatise no longer will see anything in the clouds in the future of which they will not easily understand the cause, or which will lead them to wonder." [23] Yet even as he penned these words there were questions on the radius of the rainbow which he himself was unable to answer. For one thing, he incorrectly denied the possibility of more than two rainbows at a time; [24] but his successors, notably Halley and Bernoulli, calculated the radii of innumerably many rainbows predictable under the Cartesian geometrical theory,[25] not only for drops of water, but for spherical drops of any index of refraction. Rainbows of third and fourth order have been seen in nature, and more than a dozen and a half simultaneous rainbows have been observed under laboratory conditions.

With respect to the width of the rainbow band Descartes also was wrong. Only with Newton's discovery of dispersion was it possible to explain how the radius of the rainbow varies with color. For the primary bow Newton calculated a radius of 40°17′ for the red rays and one of 42°2′ for the violet; for the secondary arc he found radii of 50°57′ and 54°7′ for the violet and red rays respectively.[26] More significant still was the erroneous impression of Descartes that, whether the drops "are larger or smaller, the appearance of the bow is not changed in any way." [27] Even in his day it was known that the distribution of colors in the rainbow was not always the same; and during the eighteenth century it was realized that the radius of the bow is not nearly so invariable as Descartes and Newton had supposed. For a very fine mist the radius

[19] *Theoremata*, pp. 68f, 73; *Photismi* (Crew), pp. 91f, 99.

[20] More accurately, he placed its limits between 51°37′ and 54°. See his *Oeuvres*, *6*, 336, 340, 706–708.

[21] Even today, however, these anticipations continue to be overlooked, with the result that Descartes continues to be regarded as "the first to report the angles [of the rainbows] correctly." See Laurence J. Lafleur, "Descartes' role in the history of science," *The Scientific Monthly*, *71* (1950), 11–14. See p. 13.

[22] *Oeuvres complètes*, *10*, 405.

[23] *Oeuvres*, *6*, 366, 720.

[24] *Oeuvres*, *6*, 342–343, 709.

[25] Edmund Halley, "De iride . . . dissertatio geometrica," *Philosophical Transactions*, *22* (1700–1701), 714–725; Jean Bernoulli, *Opera omnia* (4 vols., Lausannae and Genevae, 1742), *4*, 197–203.

[26] Sir Isaac Newton, *Opticks* (reprinted from the 4th ed. of 1720, London, 1931), pp. 168–178.

[27] *Oeuvres*, *6*, 325, 700.

may be as much as half a dozen degrees smaller than the one which Descartes had measured and calculated.[28] This fact led Young to question the whole Cartesian theory of the rainbow, with the result that the geometrical explanation gave way to a physical theory based upon the wave theory of light.[29] So thoroughly did Airy and others [30] investigate the rainbow problem from the point of view of interference that, whereas Descartes once confidently calculated the size of the rainbow from the geometry of a spherical drop, it now became possible, inversely, to calculate from the observed radius and characteristics of a given rainbow,[31] the size of the drops producing it! Had Descartes been a better historian of science, he would have realized that his work was neither the first nor the last significant contribution on the radius of the rainbow.

[28] For accurate measures of the radius during the nineteenth century, see J. G. Galle, "Measurements of the rainbow," *Philosophical Magazine* (3), *26* (1845), 279–280.

[29] Thomas Young, *A course of lectures on natural philosophy* (2 vols., London, 1807), *1*, 470f.

[30] D. Hammer, "Airy's theory of the rainbow," *Journal of the Franklin Institute*, *161* (1903), 335–349.

[31] O. D. Chwolson, *Traité de physique* (transl. from the Russian and German by E. Davaux, new ed., *2*, Paris, 1906), p. 553f.

Galileo, Hobbes, and the Circle of Perfection

BY SAMUEL I. MINTZ *

THE Circle of Perfection has been the subject of a recent study by Professor Marjorie Nicolson,[1] and I should like to add a note to her valuable work. Miss Nicolson has demonstrated that Elizabethan cosmology was "most often interpreted in terms of the circle — a circle that most [Elizabethans] believed actually existed in the perfect spheres of the planets, in the sphere of the globe, in the round head of man. This was more than analogy to them; it was truth. God had made all things in the universe, the world, and the body of man as nearly circular as grosser natures would allow." [2] The idea is, of course, ultimately derived from the ancients, but it was never more pervasive than during the Renaissance, when it was viewed as a corollary to the familiar doctrine of correspondences or analogies between the macrocosm and microcosm. In the earlier seventeenth century, the circle was a staple of poetic imagery. It was also viewed with varying degrees of reverence by men of science: Miss Nicolson refers particularly to Kepler, Harvey, and Gilbert. To this list she might have added the name of Hobbes.

In the second part of *De Corpore* (1655), Hobbes introduces the principle of inertia, which he adopted with no significant variation from Gassendi and Descartes. "Whatsoever is moved," he writes, "will always be moved on in the same way and with the same velocity, except it be hindered by some other contiguous and moved body." [3] In the third part of *De Corpore* he presents a geometrical reduction of Copernicus' two annual motions of the earth to one simple circular motion, and then considers, at somewhat greater length, a few of the mechanical consequences of a simple circular

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[1] Marjorie H. Nicolson, *The Breaking of the Circle* (Evanston, 1950).

[2] *Ibid.*, p. xx.

[3] Thomas Hobbes, *The English Works*, W. Molesworth, ed. (London, 1839), *1*, 125.