Background from Algebra, Precalculus, and Calculus I which you are required to get down cold and background from Calculus II and Linear Algebra which you are required to be comfortable using

Algebra

Quadratic equations:

(a) Roots of $ax^2 + bx + c = 0$ are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \equiv \frac{-(b/2) \pm \sqrt{(b/2)^2 - ac}}{a}.$$

(b) Special cases of the above:

- $c = 0$: then $x_1 = 0$, $x_2 = -b/a$;
- $b = 0$: then $x_{1,2} = \pm\sqrt{-c/a}$.

(c) Graphs of $y = ax^2 + bx + c \equiv a(x - x_1)(x - x_2)$ for both $a > 0$ and $a < 0$.
If you do not remember these graphs, use Mathematica or any other software to plot them.
For example, in Mathematica the command is:

```
Plot[3*(x-2)*(x-6), {x,-1, 10}]
```

Exponential functions:

$$a^{m+n} = a^m \cdot a^n, \qquad a^{m-n} = a^m/a^n;$$

$$a^{mn} = (a^m)^n = (a^n)^m \,.$$

Precalculus

Logarithms:

$$\ln(ab) = \ln a + \ln b, \qquad \ln a^n = n\,\ln a, \qquad \ln(a/b) = \ln a - \ln b, \qquad \ln\left(\frac{1}{a}\right) = -\ln a\,.$$

Exponentials and Logarithms:

$$e^{\ln a} = a, \qquad e^{\ln a + \ln b} = ab, \qquad e^{f(t)+C} = e^{f(t)} \cdot e^C, \qquad e^{a\,\ln t} = t^a.$$

Graphs of exponentials, sines, and cosines:

You must know:

- What graphs of e^{at} look like for both $a > 0$ and $a < 0$ for $-\infty < t < \infty$.
  Do `Plot[Exp[x], {x,-0.5, 1.5}]` etc. if you do not remember.
- How graphs of e^{2t} and e^{t} differ (which one goes higher/lower):
  - for $t > 0$;
  - for $t < 0$.
- How graphs of e^{-2t} and e^{-t} differ (which one goes higher/lower):
  - for $t > 0$;
  - for $t < 0$.
- Where graphs of $\sin t$ and $\cos t$ have zeros and maxima and minima.
- How graph of $\sin 2t$ and $\sin(t/2)$ differ from the graph of $\sin t$.

Basic Calculus I (must know cold)

Differentiation rules:

- $$y'(t) = f(t) \qquad \Leftrightarrow \qquad y(t) = \int f(t)\,dt\,.$$
- $$(f \pm g)' = f' \pm g';$$
- $$(c\,f(t))' = cf'(t) \qquad \text{if and only if} \qquad c = \text{const};$$
- $$(f\,g)' = f'\,g + f\,g' \qquad \text{(Product Rule)};$$
- $$\frac{d\,f[u(t)]}{dt} = \frac{df}{du} \cdot \frac{du}{dt} \qquad \text{(Chain Rule)}.$$

Derivatives and integrals of basic functions:

Everywhere below $a = \text{const}$; '$+C$' is implied for all indefinite integrals.

- $$(e^{at})' = a\,e^{at}; \qquad \int e^{at}dt = \frac{1}{a}e^{at}.$$

- $$(\sin(at))' = a\,\cos(at); \qquad \int \sin(at)dt = -\frac{1}{a}\cos(at).$$

- $$(\cos(at))' = -a\,\sin(at); \qquad \int \cos(at)dt = \frac{1}{a}\sin(at).$$

- $$\begin{cases} (t^a)' = a\,t^{a-1} \\ (\ln t)' = t^{-1}\,; \end{cases} \qquad \begin{cases} \int t^a dt = \frac{1}{a}t^{a+1}, \quad a \neq -1 \\ \int t^{-1} dt = \ln|t|\,. \end{cases}$$

Fundamental Theorem of Calculus:

$$\int_{t_0}^{t} F'(t_1)\,dt_1 = F(t) - F(t_0).$$

Of course, any "letter" will work in place of $F(t)$; e.g., $y(t)$.

Separation of Variables (no longer taught in Calculus at UVM, but belogs there):

$$\frac{dy}{dt} = \frac{M(t)}{N(y)} \qquad \Leftrightarrow \qquad \int N(y)\,dy = \int M(t)\,dt\,.$$

Calculus I / II (must feel comfortable using)

u-substitution:

For example:

$$\begin{aligned} \int e^{ax}\,dx = & \quad \Big| \quad u = ax, \quad du = a\,dx \quad \Rightarrow \quad dx = \frac{1}{a}\,du \\ \frac{1}{a}\int e^u du = & \quad \Big| \\ \frac{1}{a}e^u + C = \frac{1}{a}e^{ax} + C & \quad \Big| \end{aligned}$$

Integration by parts:

This corollary of the Product Rule will be helpful, but I do *not* expect it from memory:

$$\int f(t)g'(t)dt = f(t)g(t) - \int f'(t)g(t)dt$$

Alternative form of the same:

$$\int f(t)\,dg(t) = f(t)g(t) - \int g(t)\,df(t)\,.$$

Partial fraction expansion (PFE) :

You may need the simplest case of PFE:

$$\frac{as+b}{(s+c)(s+d)} = \frac{A_1}{s+c} + \frac{A_2}{s+d};$$

you should review your Calculus textbook on how to find the constants A_1 and A_2.

Linear Algebra (must feel comfortable using)

Solution of 2×2 and 3×3 linear systems:

While you may still solve the former systems by substitution/elimination (i.e., as you did in high school), I strongly recommend that you use the Reduced Echelon Form (a.k.a. Gauss or Gauss–Jordan elimination) algorithm to solve 3×3 systems. (Note that in the later part of the course, where you are required to solve for eigenvectors, you will need to handle systems whose solution contains "free" (i.e., arbitrary) variables.)

Singular matrices:

Definition:
A square matrix A is singular if there is a **nonzero** vector $\vec{x}$ (i.e., $\vec{x} \neq \vec{0}$) such that $A\vec{x} = \vec{0}$.
Equivalently:
A square matrix A is non-singular if the **only** solution to $A\vec{x} = \vec{0}$ is $\vec{x} = \vec{0}$.

Properties of singular / non-singular matrices:

- The linear system $A\vec{x} = \vec{b}$ is guaranteed to have a (unique) solution $\vec{x}$ for any vector $\vec{b}$ if and only if A is non-singular;
- A^{-1} exists if and only if A is non-singular;
- $\det A \neq 0$ if and only if A is non-singular (equivalently, $\det A = 0$ if and only if A is singular);
- Columns of a singular matrix are linearly dependent, and so are rows (i.e., one column is a linear combination of the other ones, and similarly for rows).

Eigenvalues λ & eigenvectors $\vec{x}$ of a square matrix A

satisfy the equation $A\vec{x} = \lambda\vec{x}$ provided that $\vec{x} \neq \vec{0}$.
You must be comfortable finding eigenvalues and eigenvectors for 2×2 matrices.