**Suggested topics for the final presentation**

**(most sources available from instructor or by searching online)**

1. Modified equation.

Possible sources (I have copies):

book by J.D. Hoffmann, Secs. 10.3, 10.7, 10.9, 12.3-12.5;

excerpts from book by Griffiths & Higham, Chaps. 13-15.

Goal: Illustrate it using the example of symplectic methods of Lec. 5.

1. Stochastic differential equations (SDEs).

Possible sources:

Tutorials by D. Higham and T. Sauer.

books: (i) “[**Numerical solution of stochastic differential equations / Peter E. Kloeden, Eckhard Platen;**](https://primo.uvm.edu/primo-explore/fulldisplay?docid=UVM_VOYAGER653174&context=L&vid=UVM&lang=en_US&search_scope=All_Catalog&adaptor=Local%20Search%20Engine&tab=default_tab&query=any%2Ccontains%2Ckloeden&offset=0) (ii) [**Numerical solution of SDE through computer experiments / Peter E. Kloeden, Eckhard Platen, Henri Schurz.**](https://primo.uvm.edu/primo-explore/fulldisplay?docid=UVM_VOYAGER658106&context=L&vid=UVM&lang=en_US&search_scope=All_Catalog&adaptor=Local%20Search%20Engine&tab=default_tab&query=any%2Ccontains%2Ckloeden&offset=0) available from UVM library

Goal: Present the Euler-Maruyama (aka simple Euler for SDEs) and Milstein methods and compare their performance for some SDE with multiplicative noise. (Note: You must present a derivation of the Milstein method.) Does your comparison confirm the statements made about the accuracies of these two methods?

1. Explicit symplectic methods of order higher than 2; e.g., the Forest—Ruth algorithm and its variants. Reference to Forest – Ruth can be found, e.g., in [**http://arxiv.org/pdf/cond-mat/0110585v1.pdf**](http://arxiv.org/pdf/cond-mat/0110585v1.pdf). Also paper by Bandrauk, Yoshida, Suzuki.

Goal: Learn basics of operator splitting methods (using Baker-Hausdorff formula).

1. Read the paper by M. Holmes and present the derivation and a couple of examples illustrating his *implicit* symplectic method. You may also experiment with how the accuracy of solving the implicit equations by Newton’s method affects the accuracy of this symplectic method.
2. Singular IVPs and BVPs (with nonsingular solutions, like the Bessel function).

Possible sources:

[**Computer solution of ordinary differential equations : the initial value problem / L.F. Shampine, M.K. Gordon.**](https://primo.uvm.edu/primo-explore/fulldisplay?docid=UVM_VOYAGER1168484&context=L&vid=UVM&lang=en_US&search_scope=All_Catalog&adaptor=Local%20Search%20Engine&tab=default_tab&query=creator%2Ccontains%2Cl.f.%20shampine%2CAND&mode=advanced&pfilter=pfilter%2Cexact%2Cbooks%2CAND&offset=0) , Chap. 12.

Must present the basics and results for Bessel functions of different order, comparing your solution with Matlab’s besselj.

If two people happen to choose this topic, then one person can do the basics described in the above source (plus see below), and the other person can address the following issue.

As you experiment with Bessel functions of increasing order, you will notice that the accuracy of the computed solution depends on how many “initial steps” one takes near 0. (This observation should be in the talk by the first speaker.) This raises the question: Given this sensitivity, how are Bessel functions actually computed in the professional software like Matlab? You can present your report on this question using, for example, two sources:

Cleve Moler’s (Matlab’s founder) [**notes on recursion relations for Bessel functions**](https://blogs.mathworks.com/cleve/2017/11/06/three-term-recurrence-relations-and-bessel-functions/);

[**The page on computation of Bessel functions**](https://dlmf.nist.gov/10.74) on NIST’s (National Institute of Standards and Technology) website. Here you will need to read sections (ii, iv, v) and explore links therein.

1. Two-component coupled parabolic PDEs: must use the block-Thomas algorithm.
2. Model pattern formation in a nonlinear reaction-diffusion-type equation.

Possible source: papers posted on the website below this link (and there are tons of papers on this topic).

Goal: Apply an explicit or implicit method for a nonlinear version of Heat equation

1. Model a **stiff** reaction-diffusion system with explanations of the choices of methods and parameters, based on stability. Application of some IMEX method.
2. Solve a parabolic equation in 2D with a mixed-derivative term and time-dependent boundary conditions by the Graig—Sneyd method. Confirm accuracy and stability as described in Lecture 15. Possible sources: find a relevant Fokker—Planck (it occurs in the theory of random processes) equation in “Handbook of stochastic methods” by C. Gardiner (I have a copy).
3. Read a paper by S. Chin on Saulyev-type methods (an alternative to ADI methods) and present his point of view on their stability.
4. Learn two finite-difference schemes for the wave equation u\_tt = u\_xx other than the simple central-difference scheme (e.g., Lax—Friedrichs, Lax—Wendroff, Beam—Warming, McCormack,…). Present their consistency and stability analyses.

Possible source: book by J.D. Hoffmann (I have a copy).

1. Relate item 1 (modified equation) to one of the schemes mentioned in the previous item (item 10).
2. Read about *discontinuous Galerkin methods*. Explain why they are needed and their main idea (and possibly a simple example). Find the material online; it should be plentiful (probably search for “tutorial discontinuous Galerkin”).
3. Read Lecture 17 and apply its method to find the soliton of equations of pulse propagation in nonlinear gratings or the nonlinear Dirac equation (I can supply the equations).
4. Efficient solution of a 2D Poisson equation (a 2D version of y’’=f) by the Block Cycling Reduction method (some kind of generalization of Thomas algorithm for 2D).

Possible source: Secs. 6.1-6.4 and 6.8 from J. Demmel, “Applied Numerical Linear Algebra” (I have a copy).

1. Numerical solution of matrix eigenvalue problems (what is the handwritten “Lecture 10,” which we do not cover).

Possible source: My Lecture 10, or secs. 3.2 and 3.3.2 of L.W. Johnson, R.D. Riess, “Numerical Analysis”.

Option 1: Do power method (very briefly), inverse power, deflation, Rayleigh quotient.

Option 2: Householder transformation.

Must do some of the problems for HW 10.