SELF-INDUCED TRANSPARENCY SOLITONS IN NONLINEAR REFRACTIVE PERIODIC MEDIA

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We obtain new nonstationary soliton-like solutions for an extended version of the classical massive Thirring model which, in nonlinear optics, describes Bragg-resonant wave propagation in a periodic Kerr medium. These solitons represent intense optical wavetrains whose envelope travels unchanged through a distributed feedback reflection filter, in spite of the fact that the mean wavelength of the soliton is in the center of the forbidden gap. The soliton group velocity may be anywhere between zero and the speed of light in the medium.

In this Letter we discuss the spatiotemporal interaction between two counterpropagating modes of the electromagnetic field inside a one-dimensional periodic nonlinear medium. Wave propagation in linear periodic structures has been studied for a long time and is relevant in a variety of fields of application, for example solid-state physics and integrated optics [1]. Whenever the nonlinearity of the material gives rise to additional light-induced gratings, new physical effects have been predicted to occur. For example, in the steady state, the intensity dependent refractive index may alter the phase matching condition and lead to optical bistability or "high" transmissivity for beams whose frequency lies in the otherwise forbidden gap of the grating [2,3]. In the nonstationary case, earlier studies have indicated that propagation in nonlinear distributed feedback structures may exhibit a host of dynamical behaviors such as instabilities, chaos, pulse compression and solitary waves [4–7], and most recently [8], a new class of optical solitons was obtained. As we will show, they correspond to a particular form of the two-parameter soliton-like family presented here. This more general class of solutions gives us, as we will see, answers to some issues pointed out in ref. [8] such as relation to previous work and stability properties of the solutions.

In this work we point out that the nonstationary interaction is represented here by equations which are a generalization (by inclusion of self-phase modulation, or SPM) of the classical massive Thirring model (MTM) of field theory [9]. This model has been shown to be completely integrable by means of the inverse scattering transform [10,11]. This generalization is different from the extension considered in ref. [12] where only for a particular choice of parameters where the equation was gauge invariant to the integrable model, soliton-like solutions were obtained using Bäcklund transformations. Our study also differs from that of the polarization domains in a uniform nonlinear medium [13], where not only there is no linear coupling, but the self-phase

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modulation effect was ignored. The approximations made, reduced the model to an integrable system of an anisotropic chiral field on group O(3).

Here we shall derive a family of nonstationary soliton-like solutions of the full problem (i.e., with SPM) which maintain some of the general physical properties of the MTM solitons. In particular, these waves exhibit stability under collisions and satisfy the integrability conditions for a Lax scheme (in the form given by Kaup and Newell [11]) which, however, does not imply integrability of the full problem.

We write the total electric field in a waveguide as the sum of two counterpropagating modes,

$$E(R,Z,T) = [E_1(Z,T)e^{i\beta Z}+E_2(Z,T)e^{-i\beta Z}]E_{Tr}(R)e^{-i\omega T},$$

where ω is the mean frequency, and $E_{Tr}(R)$ is the common transverse ($R=(X,Y)$) mode field distribution. Two mechanisms concur in the coupling of the above fields. First, suppose that a matched index grating has been written into the medium: The linear refractive index $n=n_0+n_1(Z)$ is a periodic function of Z, with a Fourier component of spatial period Λ satisfying the Bragg condition $\Lambda=\pi/\beta$. Additionally, interaction occurs between the field and the medium which reacts back to the field through a third-order polarizability $P_{NL}=\chi\epsilon_0 EEE^*$. The resulting coupled equations read

$$\partial_T E_1+V\partial_Z E_1=i\kappa E_2+i(R_1|E_1|^2+R_2|E_2|^2)E_1,$$
$$\partial_T E_2-V\partial_Z E_2=i\kappa E_1+i(R_1|E_2|^2+R_2|E_1|^2)E_2, \quad (1)$$

where $V=(\partial\beta/\partial\omega)^{-1}|_\omega=c/n$ is the group velocity of light in the material (with effective refractive index n) in the absence of mode coupling, κ is a linear coupling coefficient and $R_{1,2}$ are nonlinearity coefficients involving χ and overlap integrals of the modal distribution E [2]. In dimensionless units, eqs. (1) read

$$\partial_t e_j=i\frac{\delta H}{\partial e_j^*}, \quad \partial_t e_j^*=-i\frac{\delta H}{\delta e_j}, \quad j=1,2, \quad (2)$$

where

$$H=\int dz\,[\tfrac{1}{2}i(e_1^*\partial_z e_1-e_1\partial_z e_1^*-e_2^*\partial_z e_2+e_2\partial_z e_2^*)$$
$$+e_2e_1^*+e_2^*e_1\pm|e_1|^2|e_2|^2\pm\tfrac{1}{2}\sigma(|e_1|^4+|e_2|^4)].$$

In the above Hamiltonian, upper and lower signs hold for a focusing or defocusing nonlinearity, respectively. Note that, by interchanging z and t, eqs. (2) represent the coupling between two copropagating waves with different group velocity (e.g., two orthogonal polarization modes) [14].

In the limit $\sigma=0$, eqs. (2) reduce to the Thirring model; we now present a generalization of the one-soliton solution for the general case with $\sigma\neq 0$, using three different approaches. In doing this, we intend to highlight the similarities with the MTM solitons and to make a connection with previous work.

Let

$$e_{1(2)}=\alpha\psi_{1(2)}(z,t)\exp[i\theta(\xi)], \quad (3)$$

where $\psi_{1(2)}$ is the one-soliton solution of the Thirring model [10,11], α is a constant to be determined as well as the phase $\theta(\xi)$ where $\xi=(z-vt-z_0)/(1-v^2)^{1/2}$, $|v|<1$. Substitution of (3) in eqs. (2) gives two equations for θ,

$$\frac{d\theta}{d\xi}=\left(\sigma\alpha^2\frac{1+v}{1-v}+(\alpha^2-1)\right)$$
$$\times\sin^2 Q\,|\mathrm{sech}(\xi\sin Q-\tfrac{1}{2}iQ)|^2, \quad (4a)$$

$$\frac{d\theta}{d\xi}=-\left(\sigma\alpha^2\frac{1-v}{1+v}+(\alpha^2-1)\right)$$
$$\times\sin^2 Q\,|\mathrm{sech}^2(\xi\sin Q-\tfrac{1}{2}iQ)|^2. \quad (4b)$$

The condition that the right hand side of (4a), (4b) should be the same for them to be consistent determines the value

$$\alpha=\left(\frac{1-v^2}{(1-v^2)+\sigma(1+v^2)}\right)^{1/2}.$$

Finally, upon substitution of α in (4a) or (4b) and an integration, we are able to determine θ and thus e_1 and e_2; they read

$$e_1=\alpha\left(\frac{1+v}{1-v}\right)^{1/4}\sin Q\exp\left(\mp i\frac{t-vz}{\sqrt{1-v^2}}\cos Q\right.$$
$$\left.+i\phi+i\theta(\xi)\right)\mathrm{sech}(\xi\sin Q\mp\tfrac{1}{2}iQ), \quad (5)$$

$$e_2 = \mp \alpha \left(\frac{1-v}{1+v}\right)^{1/4} \sin Q \exp\left(\mp \mathrm{i} \frac{t - vz}{\sqrt{1-v^2}} \cos Q + \mathrm{i}\phi + \mathrm{i}\theta(\xi)\right) \operatorname{sech}(\xi \sin Q \pm \tfrac{1}{2}\mathrm{i}Q) , \qquad (6)$$

with

$$\theta = \mp \frac{4\sigma v \alpha^2}{1-v^2} \operatorname{arctg}[\,|\operatorname{cotg} \tfrac{1}{2}Q|\, \coth(\xi \sin Q)] ,$$

$0<Q<\pi$, and $-1<v<1$.

In expressions (5), (6), upper and lower signs hold in the case of focusing or defocusing nonlinearity, respectively. As in the one soliton solution of the Thirring model, the two components of the present grating self-transparency (GST) solitons are characterized by two parameters Q and v which determine the pulse width and velocity of propagation. When $Q=\frac{1}{2}\pi$ one can show that (5), (6) reduce to the "slow Bragg solitons" obtained in ref. [8]. A second interesting limit is when $Q\to 0$ and $|v| \ll 1$; then (5), (6) reduce to the NLS one-soliton solution. That is, for slow small amplitude broad pulses, the description given in ref. [6] correctly applies. Finally, the limit $Q\to\pi$ gives finite plane wave solutions of (2). Notice that none of the last two limits can be obtained from the solutions given in ref. [8]. A deeper discussion of all types of solutions of (2) and their properties will be given elsewhere [15].

Fig. 1 illustrates the group velocity dependence of the intensity ratio between the backward and forward components e_2 and e_1 (solid line) along with the common normalized spatial width (dot-dashed line). As can be seen, in the stationary case the two envelope components of the soliton are equally intense. On the other hand, whenever the group velocity v approaches the velocity of light in the medium, the component with opposite signs of phase and group velocity becomes negligibly small. Correspondingly, the width of the hyperbolic secant envelope narrows down to zero. In the figure, we report the dimensionless quantity

$$w = \frac{\kappa n}{Wc} = (1-v^2)^{1/2} ,$$

which, in the case $Q=\frac{1}{2}\pi$, yields in real units a spatial width of the hyperbolic secant equal to $1/W$. In general, this width is $(W \sin Q)^{-1}$.

Numerical simulations indicated that eqs. (5), (6) yield physically stable and robust solutions. More specifically, even in cases where the initial conditions did not closely match the shape of e_1 and e_2, the computed envelopes did indeed evolve into one member of the family after losing some power into radiation (see fig. 2: in these and in the following

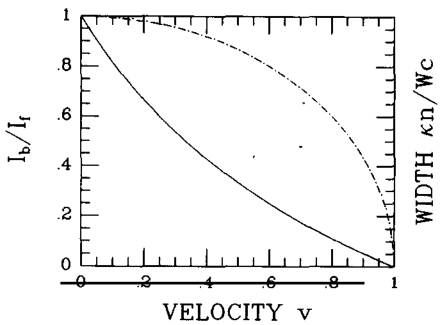

Fig. 1. Intensity ratio of backward to forward components (solid line) and normalized spatial width (dot-dashed line) of fundamental solitons pulses, versus absolute value of group velocity v.

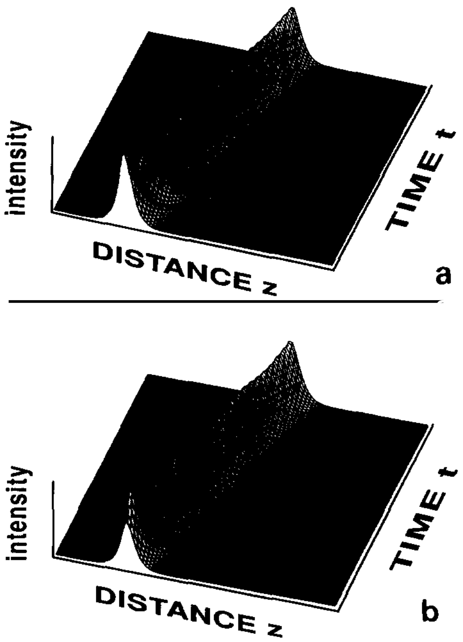

Fig. 2. Evolutions of (a) $|e_1|^2$ and (b) of $|e_2|^2$, for an initial condition that does not exactly match the expressions for the family of self-transparency solitons. After some radiation losses, the field still evolves into a member of the family.

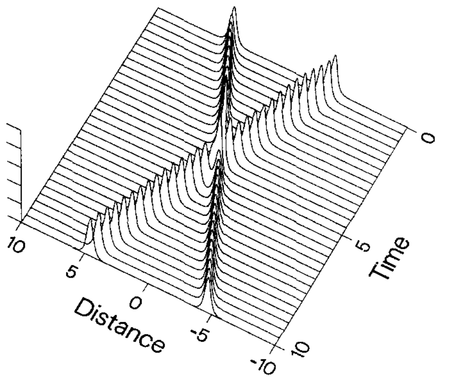

Fig. 3. Fast collision of two GST solitons. We display here sums of the envelope intensities $|e_1|^2+|e_2|^2$ for two counterpropagating solitons with velocities v equal to ± 0.9. No apparent distortion in the shapes or velocities is observed, while only a small position shift occurs.

simulations, $\sigma=0.5$). We have also observed that when two GST pulses collide, their shapes and velocities upon emerging from the interaction are almost unaffected. Fig. 3 shows the collision of two fast ($v=\pm 0.9$) equally intense self-transparency solitons, whose initial shapes were computed from eqs. (5), (6) with $\cos Q=0$. The interaction time is relatively short, so that the pulses pass through each other with little distortion. On the other hand, fig. 4 reports the collision between relatively slow ($v=\pm 0.1$) solitons: in this case, even though during the time of collision the pulses get dramatically reshaped, two pulses still emerge from the interaction with a substantial time shift but little change in velocity. Note however, the small intensity oscillation superimposed onto the outgoing pulses. All these properties indicate that indeed the solutions (5), (6) essentially share the stability properties of true solitons. The discussion which follows intends to explore this point in further depth.

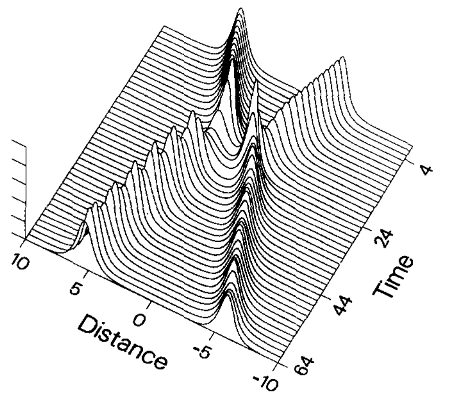

Fig. 4. As in fig. 3, with velocities $v=\pm 0.1$. A longer interaction takes place and a slight oscillation is superimposed on the emerging pulses.

We shall see below that one may obtain the nonstationary or propagating soliton solutions of the Thirring model by simply invoking the Lorentz invariance of eqs. (2). This observation also applies to the general case of interest here. To be more specific, one finds that if $\psi_{1(2)}$ is of the form

$$\psi_{1(2)}(z,t)=K_{1(2)}\hat{\psi}_{1(2)}(\xi,Q)\times\exp\left(\mp\mathrm{i}\cos Q\frac{t-vz}{\sqrt{1-v^2}}\right),$$

where

$$K_1=\frac{1}{K_2}=\left(\frac{1+v}{1-v}\right)^{1/4},$$

then the equations for $\hat{\psi}_{1(2)}$ become

$$-\mathrm{i}\hat{\psi}_{1\xi}+\hat{\psi}_2\pm\cos Q\,\hat{\psi}_1+|\hat{\psi}_2|^2\hat{\psi}_1\pm\sigma\frac{1+v}{1-v}|\hat{\psi}_1|^2\hat{\psi}_1=0, \quad (7)$$

$$\mathrm{i}\hat{\psi}_{2\xi}+\hat{\psi}_1\pm\cos Q\,\hat{\psi}_2+|\hat{\psi}_1|^2\hat{\psi}_2\pm\sigma\frac{1-v}{1+v}|\hat{\psi}_2|^2\hat{\psi}_2=0. \quad (8)$$

Note that the addition of the self phase modulation term makes these equations to be v dependent, nevertheless as we will see, SPM only brings a correction into the phase and a new rescaling of the field. These equations, as in the $\sigma=0$ case, have an invariant, $|\hat{\psi}_1|^2-|\hat{\psi}_2|^2=0$, therefore $\hat{\psi}_{1,2}=f(\xi)\times\exp[\mathrm{i}\theta_{1,2}(\xi)]$ and the problem reduces to solving for the functions f, θ_1 and θ_2.

A further simplification can be achieved if we define $2\mu=\theta_1-\theta_2$. We obtain for μ and f the following equations,

$$\frac{\mathrm{d}\mu}{\mathrm{d}\xi}=\pm\cos Q+\cos 2\mu\pm(1/\alpha^2)f^2,$$

$$\frac{\mathrm{d}f}{\mathrm{d}\xi}=\sin 2\mu f, \quad (9)$$

where one must read two systems of equations, each corresponding to one choice of signs. After solving for μ and f, θ_1 and θ_2 are solved by quadrature. For the positive nonlinearity case, the equations were first solved by Chang, Ellis and Lee [16] in their study of fermion confinement in a chiral-symmetric theory in 1+1 dimensions. Their confined and time independent solutions to the classical massive Thirring model were obtained before the integrability of the model was determined. In ref. [3], solutions of eqs. (9) were given for both choices of signs of the nonlinearity. In our case, we obtain from eqs. (9) the two solutions given in (5), (6). It is important to point out that, when ξ is replaced by z, eqs. (9) were also derived by Mills and Trullinger in their description of stationary localized waves (occurring in the forbidden frequency zones thus baptised gap solitons) inside nonlinear superlattices [3].

We conclude by showing how the coupled equations (1) are in some sense the integrability condition similar to that obtained in refs. [10,11]. We shall follow the approach of the second reference, although both are equivalent. For the sake of conciseness, we shall restrict our treatment to the positive nonlinearity case. Let $x=(t+z)/2$ and $\rho=(z-t)/2$ and consider the system

$$\partial_x v_1 + \mathrm{i}\zeta^2 v_1 = \zeta q v_2\,, \tag{10a}$$

$$\partial_x v_2 - \mathrm{i}\zeta^2 v_2 = \zeta q^* v_1\,, \tag{10b}$$

$$\partial_\tau v_1 - \mathrm{i}\left(\frac{1}{4\zeta^2} - \frac{1}{\alpha^2}|e_1|^2\right)v_1 = -\frac{1}{\sqrt{2}\,\alpha} e_1 \mathrm{e}^{-\mathrm{i}\mu-\mathrm{i}\theta} v_2\,, \tag{11a}$$

$$\partial_\tau v_2 + \mathrm{i}\left(\frac{1}{4\zeta^2} - \frac{1}{\alpha^2}|e_1|^2\right)v_2 = \frac{1}{\sqrt{2}\,\alpha} e_1^* \mathrm{e}^{\mathrm{i}\mu+\mathrm{i}\theta} v_1\,, \tag{11b}$$

where

$$q = \frac{\sqrt{2}}{\alpha} e_2 \exp\left(\mathrm{i}\int_x^\infty |e_2|^2\,\mathrm{d}x - \mathrm{i}\theta\right)$$

and θ for the moment is an unknown function which may depend on $|e_1|$ and $|e_2|$. If $\alpha=-1$, $e_{1(2)}=\psi_{1(2)}$ and $\theta=$const, then the equations for the Thirring model are the consistency condition $v_{x\tau}=v_{\tau x}$ of (10), (11) and in particular if in the scattering problem (10) one substitutes as the potential the corresponding one-soliton solution, a single eigenvalue $\zeta=|\zeta|\exp[\mathrm{i}\arg(\zeta)]$ (where $\arg(\zeta)=\frac{1}{2}Q$ and $|\zeta|$ is given by $v=(|\zeta|^{-2}-|\zeta|^2)/(|\zeta|^{-2}+|\zeta|^2)$), exists [10]. In general, the equations that result from the consistency condition read

$$\partial_\tau e_2 = -\mathrm{i}e_1 - \mathrm{i}[(2/\alpha^2-1)|e_1|^2 - \theta_\tau]e_2\,, \tag{12a}$$

$$\partial_x e_1 = \mathrm{i}e_1 + \mathrm{i}(|e_2|^2 + \theta_x)e_1\,. \tag{12b}$$

Consider now the additional conditions on θ,

$$\theta_x = \sigma|e_1|^2\,,$$

$$\theta_\tau = -2(1-1/\alpha^2)|e_1|^2 - \sigma|e_2|^2\,. \tag{13}$$

While a solution of eqs. (12), (13) would also satisfy the original equations (2), it is not necessarily true that for every solution of (2) there is a θ satisfying (13). Nevertheless, for the GST solitons (5), (6) there is a θ satisfying (13) thus eqs. (12), (13) and (2) are equivalent. In this sense we may say that the class of solutions (5), (6) arise from a Lax pair formalism where the same eigenvalue of the integrable ($\sigma=0$) case occurs.

The interesting behavior of the present GST solitons suggests directions for the continuation of the present study. In the context of nonlinear fiber optics, some specific applications and a more detailed analysis of the behavior of GST solitons in terms of real parameters will appear in forthcoming papers [14,15]. The physical relevance of the present solutions to the classical field model (2) is, however, likely to extend beyond the context of nonlinear optics. From the theoretical point of view, a search for a possible Bäcklund transformation and a Painlevé analysis would be in order for testing the integrability of the equations. We also plan to address the possible extension of higher order MTM solitons [17] to the present case.

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