Effect of a Raman Co-Pump's RIN on the BER for Signal Transmission Using ON–OFF Keying Modulation Format

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***Abstract*—This paper theoretically studies the effect of the relative intensity noise (RIN), which is transferred to the signal from a Raman co-pump, on the bit-error rate (BER). We show that a given amount of the transferred RIN degrades the BER by about an order of magnitude more than does the same amount of the amplified spontaneous emission (ASE) added to the signal. We investigate how this effect depends on such parameters as the duty ratio of the signal pulses and on the amount of the ASE that has been added to the output signal along the transmission line.**

***Index Terms*—Noise, optical fiber amplifiers, optical fiber communication.**

I. Introduction

DISTRIBUTED Raman amplification has become widely used in fiber-optic transmission systems owing to its ability to provide a broader and flatter gain, as well as to generate less amplified spontaneous emission (ASE) noise, than erbium amplifiers. The backward-pumping scheme, where the Raman pump and the signal propagate in opposite directions, has received most of the attention, even though the forward-pumping scheme, where the pump and signal copropagate, is known to generate less ASE [1]. One reason for the latter scheme being less popular is that such detrimental effects as the signal–pump–signal crosstalk [2]–[4] and the transfer of relative intensity noise (RIN) from the pump to the signal are much stronger in the forward-pumping scheme than in the backward-pumping one.

In [5], an analytical formula for the efficiency of the transfer of the pump modulation to the signal was obtained, in the undepleted pump approximation, as a function of the walk-off parameter between the signal and the pump. (See also [6], where the case of very strong pump depletion was studied numerically.) The results of [5] allow one to calculate the amount of RIN that is transferred from a pump to the signal, assuming that the RIN spectral density does not evolve inside the fiber. Furthermore, assuming Gaussian statistics (in time) of the transferred noise, the authors of [5] calculated a bit-error-rate (BER) penalty associated with the RIN transfer. However, neither of these two previously mentioned assumptions can be used to accurately describe the RIN of multimode lasers, which are currently used as Raman pumps. First, the RIN spectral density in a given bandwidth may change by several orders of magnitude due to phase-to-amplitude noise conversion via fiber dispersion (see, e.g., [7]–[9] and references therein). Second, the statistics of a multimode pump's noise is not Gaussian [10], and moreover, the RIN transferred to the signal is *multiplicative* noise (see, e.g., [11]), in contrast with the ASE, which is *additive* noise. Therefore, the assumption of the Gaussian probability density (PD) for the signal corrupted by the transferred RIN may lead to an error when estimating the BER, which is very sensitive to the shape of the PD "tails."

In this paper, we derive an equation for the PD of the detected power of the signal corrupted by noise transferred from a Raman co-pump. More specifically, we consider the following problem. Suppose that at the end of a long-haul transmission system, the amount of the pump's RIN transferred to the signal is X mW, and the amount of the ASE (in the receiver bandwidth) is Y mW. How will the BER in this case be different from that in a case where $(X+Y)$ mW of the ASE only (i.e., no transferred RIN) is added to the input signal?

Several simplifications need to be made when carrying out the analysis. First, we consider only a linear channel, i.e., neglect the inline interaction between the signal and the noise. This assumption is reasonably well satisfied in modern terrestrial (i.e., up to 4000-km-long) transmission systems operating at both 10 and 40 Gb/s. Second, we consider detection at the receiver to be performed by an idealized matched filter (see, e.g., [12]). If $(s_0(t)+\mathcal{N}(t))$ is the input to such a filter, where s_0 and $\mathcal{N}$ are the electric fields of the uncorrupted signal and the noise, respectively, then the matched filter's output is $s_0(t)\cdot(1+\mathcal{N}_{s_0})$, where $\mathcal{N}_{s_0} = \int_{-\infty}^{\infty} s_0^*(t)\mathcal{N}(t)dt/\|s_0\|$, $\|s_0\|^2 = \int_{-\infty}^{\infty}|s_0(t)|^2\,dt$. Although this oversimplified detection model does not allow one to study dependence of the BER on such parameters as the shape and bandwidth of the filter, it still allows one to obtain an answer to the central question, formulated in the preceding paragraph. Furthermore, it is clear that in the framework of that model, the detected power is simply $\|s\|^2 = \|s_0(1+\mathcal{N}_{s_0})\|^2$.

Finally, we emphasize that the key that allows us to carry out the analysis is the fact that the transmission system in question is long-haul and thus contains many amplified spans. In other words, our analysis is *not* applicable to a system that contains only few co-pumped spans (e.g., a festoon system). Indeed, in a single span, the input and output signal powers are related by

$$P_{s,\text{out}}(t) = P_{s,\text{in}}(t)\exp\left[-\alpha L + g_R\int_0^L P_R(z,t)\,dz\right] \quad (1)$$

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where α and L are the fiber loss and length, and g_R and P_R are the Raman gain coefficient and co-pump power. In order to calculate the BER of the signal received after one span, one needs to know the PD of the exponent on the right-hand side (RHS) of (1). The PD of $P_R(z=0,t)$ is, in principle, known [10], but calculation of a PD, or even the mean-square value, of $\int_0^L P_R(z,t)dz$ in a dispersive fiber is a very complex mathematical problem.[1] We do not solve this problem here. Instead, we notice that in a multispan system, the contribution of the transferred RIN from an individual span is small compared with the total amount of transferred RIN and, moreover, is uncorrelated with similar contributions from other spans. Comparing this situation with the Brownian motion, where the total displacement of a particle is a result of many small uncorrelated displacements, we are able to derive a diffusion-type equation, or the Fokker–Planck equation (FPE), for the PD of the signal corrupted by the transferred RIN accumulated over many spans. This is done in Section II of this paper.

In Section III, we present a (numerical) solution of the resulting FPE and discuss in detail a particular example. Namely, we compare the BER in several different cases, all of which have the same total amount of the ASE but different amounts of the transferred RIN. In Section IV, we present conclusions of this study.

II. Derivation of the FPE

We begin by writing a propagation equation for the signal electric field $s(z,t)$ (see, e.g., [11])

$$is_z + \gamma(2-\chi)sP_R + \left(\gamma_R - \frac{i}{2}g_R\right)sP_R = -i\frac{\alpha}{2}s + in. \quad (2)$$

Here, γ is the fiber's nonlinearity coefficient, $\chi = 0.18$ [14] is the relative contribution of the Raman effect to the *instantaneous* nonlinear response of the fiber, γ_R is the contribution to the total nonlinearity due to the resonance part of the Raman response, and n is the ASE. In accordance with our assumption that detection of the signal is performed by an idealized matched filter, we take the temporal shape of the noise term to coincide with that of the signal (see the definition of $\mathcal{N}_{s_0}$ in Section I). Then, the peak value of $|n|^2$ is the detected ASE power in the receiver bandwidth. Furthermore, in (2), we have omitted the term $s|s|^2$ in comparison with the much greater term sP_R. We also omitted the dispersive term, which is not expected to change the statistics of the noise. From the way (2) was derived in [13], it is clear that it is valid when one can neglect pump fluctuations at frequencies higher than 1 THz. When considering the transfer of RIN from the co-pump to the signal, that assumption is justified, because the RIN is transferred efficiently only within a much narrower bandwidth of about 1 GHz or less [5]. Equation (2) can be rewritten in the form

$$s_z = \frac{g_R}{2}(1+i\kappa)P_R s - \frac{\alpha}{2}s + n \quad (2a)$$

where $\kappa = -2[\gamma(2-\chi)+\gamma_R]/g_R$. For frequency separations between the pump and the signal corresponding to the maximum Raman gain, $\gamma_R \ll g_R$ and $g_R/\gamma \approx 0.5$ [13], whence $\kappa \approx -7$. In what follows, it is convenient to write $P_R = \bar{P}_R + \delta P_R$, where $\bar{P}_R$ is the time-average, constant pump power and fluctuations δP_R occur due to the pump RIN. Then, neglecting pump depletion, one has the solution of (2a) in the form

$$\begin{aligned} s(z) &= s(0)\exp\left[\int_0^z f(z')\,dz'\right] \\ &\quad + \int_0^z n(z')\exp\left[\int_{z'}^z f(z'')\,dz''\right]dz' \\ f(z) &= \frac{1}{2}[(g_R\bar{P}_R(z)-\alpha) + i\kappa g_R\bar{P}_R(z) \\ &\quad + (1+i\kappa)g_R\delta P_R(z)]. \end{aligned} \quad (3)$$

Let z_k denote the end of the kth span and assume, without loss of generality, $\int_{z_{k-1}}^{z_k}(g_R\bar{P}_R(z)-\alpha)dz = 0$, i.e., that the fiber loss is exactly compensated by the time-average gain at the end of each span. In addition, we denote

$$\begin{aligned} \bar{\phi}_k &= \frac{\kappa}{2}\int_{z_{k-1}}^{z_k} g_R\bar{P}_R(z')\,dz' \\ \Delta G_{R,k} &= \int_{z_{k-1}}^{z_k} g_R\delta P_R(z')\,dz'. \end{aligned}$$

Then, $\int_0^{z_m} g_R\delta P_R(z)\,dz = \sum_{k=1}^m \Delta G_{R,k}$. Note that Raman gain variations $\Delta G_{R,k}$ are independent for different k (i.e., for different spans), because the pumps in different spans and, hence, their fluctuations are generated by independent lasers. Now, let $z=z_m$ in (3) and approximate the second term on the RHS of that equation as

$$\begin{aligned} \sum_{k=1}^m \int_{z_{k-1}}^{z_k} n(z')\exp&\left[\frac{1}{2}\int_{z'}^{z_k}(g_R\bar{P}_R-\alpha)\,dz'' + i\sum_{l=k}^m \bar{\phi}_l\right. \\ &\left. + \frac{1}{2}(1+i\kappa)\sum_{l=k}^m \Delta G_{R,l}\right]dz'. \end{aligned} \quad (4)$$

(The approximation was made when rewriting the last two terms in the exponent.) Note that the ASE accumulated over the kth span is $N_k = \int_{z_{k-1}}^{z_k} n(z')\exp[\int_{z'}^{z_k}(g_R\bar{P}_R(z'')-\alpha)\,dz'']dz'$. In addition, it is intuitively clear that the nonrandom phases $\bar{\phi}_k$ do not affect the statistics of the signal. To reflect this fact, we introduce two more notations

$$\begin{aligned} \tilde{s}(z_k) &= s(z_k)\exp\left[-i\sum_{l=1}^k \bar{\phi}_l\right] \\ \tilde{N}_k &= N_k\exp\left[-i\sum_{l=1}^k \bar{\phi}_l\right]. \end{aligned}$$

Then, using expression (4), (3) can be approximately rewritten as

$$\tilde{s}(z_k) = (\tilde{s}(z_{k-1}) + \tilde{N}_k\exp[i\bar{\phi}_k])\exp\left[\frac{1}{2}(1+i\kappa)\Delta G_{R,k}\right]. \quad (5)$$

Using now the assumption, stated previously, that the transmission system contains many spans, and the fact that $\Delta G_{R,k} \ll 1$,

[1] Incidentally, if P_R is a counter-pump, then the PD of $\int_0^L P_R(z,t)\,dz$ is nearly Gaussian (see Appendix A), and then, the PD of the output signal $P_{s,\text{out}}(t)$ is explicitly seen, from (1), to be non-Gaussian.

one can replace the difference equation (5) by a differential equation

$$\tilde{s}_z = (1 + i\kappa)\mathcal{G}\tilde{s} + \tilde{\mathcal{N}} \tag{6}$$

where $\mathcal{G} = (1/2)\Delta G_{R,k}/\Delta z_k$, and $\tilde{\mathcal{N}} = \tilde{N}_k e^{i\bar{\phi}_k}/\Delta z_k, \Delta z_k = z_k - z_{k-1}$. Since $\Delta G_{R,k}$ and $\tilde{N}_k$ are independent for different spans, (6) has the form of a Langevin equation with random sources $\mathcal{G}$ and $\mathcal{N}$, which are δ-correlated in z

$$\overline{\mathcal{G}(z)\mathcal{G}(z')} = 2D_{\mathcal{G}}\delta(z - z') \tag{7a}$$

$$\overline{\mathcal{N}(z)\mathcal{N}^*(z')} = 4D_{\mathcal{N}}\delta(z - z'), \quad \overline{\mathcal{N}(z)\mathcal{N}(z')} = 0 \tag{7b}$$

where the overbar denotes, as before, averaging over time.

Let us now emphasize the main difference that exists between the original equation (2a) and its corollary, the approximate equation (6). In (2'), the random source $P_R = \bar{P}_R + \delta P_R$ is *not* δ-correlated in z, which prevents one from deriving an FPE for the signal's PD directly from that Langevin equation. The reason why $\delta P_R(z,t)$ is not δ-correlated in z is that at the beginning of the span (i.e., at the output of the Raman pump laser), the pump fluctuations $\delta P_R(0,t)$ are not δ-correlated in time (see, e.g., [10]), and those correlations at different instances in time are transformed into correlations at different locations along the fiber as the pump propagates. The critical advantage that we achieved by approximately rewriting (2a) in the discrete form (5) is that the noise sources $\Delta G_{R,k}$ are uncorrelated from one span to another. Then, by going from the discrete equation (5) to its continuous version (6), which is justified if the system contains many spans, we arrived at a Langevin equation with noise sources that are δ-correlated in z. Equation (6) is used later to derive a FPE for the PD of the detected signal.

We now relate the intensity $D_{\mathcal{G}}$ of the RIN source $\mathcal{G}$ to a quantity that can be measured in a testbed experiment. The contributions to the RIN and the ASE come predominantly from forward and backward pumping, respectively [1], [5]. Therefore, the amount of transferred RIN from a co-pump to a signal in a single span can be calculated by measuring fluctuations of the output signal power after this span, when no backward pumping is applied. Considering only the first term in (3), one has

$$P_{s,\text{out}}(t) = P_{s,\text{in}}(t)\exp\left[\int_0^L (g_R\bar{P}_R(z') - \alpha)\,dz' + \Delta G_R(t)\right] \tag{8}$$

where L is the length of the span and ΔG_R is the Raman gain fluctuation in this span. Since both $\Delta G_R(t)$ and $\delta P_{s,\text{in}}(t)/\bar{P}_{s,\text{in}} \equiv (P_{s,\text{in}}(t)/\bar{P}_{s,\text{in}} - 1)$ are small and independent of each other, then from (8) one obtains

$$\overline{P_{s,\text{out}}^2} - \bar{P}_{s,\text{out}}^2 \approx \overline{\delta P_{s,\text{in}}^2} + \bar{P}_{s,\text{in}}^2\overline{\Delta G_R^2} \times \exp\left[\int_0^L (g_R\bar{P}_R - \alpha)\,dz\right]. \tag{9}$$

The first term on the RHS of (9) is due to the finite optical-signal-to-noise ratio (OSNR) of the input signal and is usually known in an experiment. Therefore, the intensity of the RIN-related source $D_{\mathcal{G}} \equiv \overline{\mathcal{G}^2(z)}(\Delta z_k/2) = (1/8)\overline{\Delta G_R^2}/\Delta z_k$ in (7a) can be found from measuring $P_{s,\text{out}}$ in a span where only the forward pumping is used, and then using (9).

The intensity of the ASE source $D_{\mathcal{N}} = |\bar{\mathcal{N}}|^2(\Delta z_k/4)$ can be found from, e.g., Eq. (10b) of [1]. Moreover, it is simply related to a change in the OSNR of the signal after one span, which can also be easily measured in the testbed. Indeed, from (5), the average signal powers at the outputs of the $(k-1)$th and kth spans are related by

$$\bar{P}_{s,k} - \bar{P}_{s,k-1} = \frac{1}{2}\bar{P}_{s,k-1}\overline{\Delta G_{R,k}^2} + 2\overline{|N_k|^2} \tag{10}$$

where the factor 2 in front of the last term accounts for the fact that the ASE is generated in both polarizations, and we have used $\overline{\exp[\Delta G_R]} \approx 1 + (1/2)\overline{\Delta G_R^2}$. Hence, the change in the reciprocal of the signal OSNR in the receiver bandwidth after one span equals

$$\frac{\bar{P}_{\text{ASE}}}{\bar{P}_{s,k-1}} = \frac{2\overline{|N_k|^2}}{\bar{P}_{s,k-1}} = \frac{8D_{\mathcal{N}}\Delta z_k}{\bar{P}_{s,k-1}}. \tag{11}$$

Similarly, the first term on the RHS of (10) represents the change in the noise-to-signal ratio due to the transferred RIN

$$\frac{\bar{P}_{\text{RIN}}}{\bar{P}_{s,k-1}} = \frac{\frac{1}{2}\overline{\Delta G_{R,k}^2}\bar{P}_{s,k-1}}{\bar{P}_{s,k-1}} = 4D_{\mathcal{G}}\Delta z_k. \tag{12}$$

We have related the noise intensities $D_{\mathcal{G}}$ and $D_{\mathcal{N}}$ in the Langevin (6) with quantities that can be measured in a testbed experiment. We now proceed to a derivation of an FPE corresponding to the Langevin equation (6). Let $\tilde{s} = re^{i\varphi}$, where r and $\varphi + \sum_{l=1}^{k}\bar{\phi}_l$ are the magnitude and phase of the signal's electric field after the kth span, and let $\rho(r,\varphi)$ be the probability density for these quantities. Following the standard prescription of the derivation of the FPE (see, e.g., [11]) in the Stratanovich form and upon performing a considerable amount of straightforward algebraic calculations, we obtain the following FPE for $\rho(r) = \int_0^{2\pi} d\varphi\rho(r,\varphi)$

$$\frac{\partial\rho}{\partial z} = D_{\mathcal{N}}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left[(1 + \mathcal{D}r^2)\rho\right]\right] \tag{13}$$

where $\mathcal{D} = D_{\mathcal{G}}/D_{\mathcal{N}}$. Note that the evolution of $\rho(r)$ does not depend on the parameter κ; this fact does not appear to be obvious from the Langevin equation (6). Let us also note that when the ASE is the only source of noise, i.e., when $\mathcal{D} = 0$, then (13) with the initial condition

$$\rho(r, z = 0) = \frac{1}{r}\delta(r - r_0), \quad r_0 > 0 \tag{14}$$

has the solution (see Appendix B):

$$\rho(r,z)|_{\mathcal{D}=0} = \frac{1}{2D_{\mathcal{N}}z}\exp\left[-\frac{r^2 + r_0^2}{4D_{\mathcal{N}}z}\right]I_0\left(\frac{rr_0}{2D_{\mathcal{N}}z}\right) \tag{15}$$

where $I_0(x)$ is the zeroth-order Bessel function of imaginary argument.

When the transferred RIN is present, i.e., for $\mathcal{D} > 0$, we were unable to find an analytical solution of (13) and, therefore, resolved to numerical solution. In Section I, we explained that within the idealized matched-filter model that we consider here, the quantity being detected by the receiver is the signal power $P_s = r^2 \equiv w$. Thus, we numerically solved an equation

$$\frac{\partial \rho(w,z)}{\partial z} = 4D_{\mathcal{N}}[\mathcal{D}\rho + (1+3\mathcal{D}w)\rho' + (w+\mathcal{D}w^2)\rho''] \quad (16)$$

instead of (13); here, $\rho' = \partial\rho(w,z)/\partial w$. The details and results of this solution are described in the next section.

III. Solution of the FPE

We solved (16) using the Crank–Nicolson scheme in the domain $(w,z) \in [0, w_{\max}] \cup [0, z_{\max}]$, where $z_{\max}$ is chosen from the requirement to have a certain OSNR of the output signal [(cf. (11)] and $w_{\max}$ is just a sufficiently large positive number. We discretize this domain as $\{w_i = (i-1)h, z_j = (j-1)\Delta\}$, where $i = 1,\dots,N, j = 1,\dots,M, N = w_{\max}/\Delta + 1, M = z_{\max}/h + 1$ and denote $\rho_i^j = \rho(w_i, z_j)$. Then, the discretized version of (16) is

$$\begin{aligned}\frac{\rho_i^{j+1} - \rho_i^j}{\Delta} &= 2D_{\mathcal{N}}\Bigg\{\mathcal{D}\left(\rho_i^j + \rho_i^{j+1}\right) + (1+3\mathcal{D}w_i) \\ &\times \left[\frac{\rho_{i+1}^{j+1} - \rho_{i-1}^{j+1}}{2h} + \frac{\rho_{i+1}^j - \rho_{i-1}^j}{2h}\right] \\ &+ w_i(1+\mathcal{D}w_i)\Bigg[\frac{\rho_{i+1}^{j+1} - 2\rho_i^{j+1} + \rho_{i-1}^{j+1}}{h^2} \\ &+ \frac{\rho_{i+1}^j - 2\rho_i^j + \rho_{i-1}^j}{h^2}\Bigg]\Bigg\}, \\ &j = 1,\dots,M-1, \quad i = 2,\dots,N-1. \quad (17)\end{aligned}$$

We take the initial condition to be a discrete form of (15) with some small $D_{\mathcal{N}} z \equiv \mathcal{N}_0$, corresponding to a large but finite OSNR $\bar{P}_{s,\text{in}}/(8\mathcal{N}_0)$, of the input signal

$$\rho_i^1 = \frac{1}{2\mathcal{N}_0}\exp\left[-\frac{w_i + w_0}{4\mathcal{N}_0}\right] I_0\left(\frac{\sqrt{w_i w_0}}{2\mathcal{N}_0}\right) \quad i = 1,\dots,N. \quad (18)$$

It is convenient to normalize quantities in (17) and (18) to the signal time-average power (also, and in what follows, called *channel power*) $\bar{P}_{s,\text{in}}$. Then $1/(8\mathcal{N}_0)$ represents the input OSNR (cf. (11)), and w_0 in (18) is the ratio between the mean detected power of a given logical bit (ONE or ZERO) and the channel power. Thus, w_0 depends on the modulation format of the signal.[2] For example, for a return-to-zero pulse with 33%-duty ratio, $w_0 \approx 3$ for a logical ONE, with w_0 decreasing when the duty ratio is increasing.

The boundary conditions for (17) are specified at $w_N = w_{\max}$ and $w_1 = 0$. The former condition, for $w_{\max}$ sufficiently large, can simply be taken as

$$\rho_N^j = 0, \quad j = 2,\dots,M \quad (19)$$

which corresponds to the vanishing PD at infinity. The condition at $w_1 = 0$ is slightly more delicate. Note that the only physical requirement that should be imposed on ρ at $w = 0$ is that ρ be integrable near that point. (In the classification scheme found in [15], such a boundary condition of a FPE is called to be of the natural type.) The eigenfunction expansion for (16) near $w = 0$, analogous to the one presented in Appendix B, shows that ρ, ρ', and ρ'' must be all finite at $w = 0$. Then (16) can be approximately written as

$$\frac{\partial \rho}{\partial z} = 4D_{\mathcal{N}}(\rho e^{\mathcal{D}w})', \quad w \approx 0. \quad (20a)$$

Notice that near $w = 0$, the order of the original equation is effectively reduced from second to first, since ρ'' is finite, and thus, $w\rho'' \approx 0$. Note also that the PD *flux* through the natural boundary at $w = 0$ vanishes, as expected: $\lim_{w\to 0} w((1+\mathcal{D}w)\rho)' = 0$. Using the simplest forward-difference expression for $\partial/\partial z$ and $\partial/\partial w$, we rewrite (20a) in the discrete form

$$\rho_1^{j+1} = \rho_1^j\left(1 - \frac{4D_{\mathcal{N}}}{h}\right) + \frac{4D_{\mathcal{N}}\Delta}{h}\rho_2^j e^{\mathcal{D}w_2}. \quad (20b)$$

This is the bondary condition at $w_1 = 0$; note that $\rho_{1,2}^1$ are known from (18). We now solve the finite-difference system (17) with the initial condition (18) and boundary conditions (19) and (20b), using the linear algebra packages of Matlab. At each step in z, we monitor that $\sum_{i=1}^N \rho_i^j = 1$ within the accuracy of the numerical method.

Typical graphs of the resulting PDs for the logical ONE and logical ZERO are shown in Fig. 1. For the case shown in that figure, we use the following set of parameters. We assume that the signal is encoded in 33%-duty cycle Gaussian pulses having a 13-dB extinction ratio between the ZEROs and ONEs; this leads to $w_0^{\text{ONE}} \approx 3$ (see text after (18)) and $w_0^{\text{ZERO}} \approx 3/20$ for the initial PDs of the ONE and ZERO, respectively. We further specify that the output OSNR in the receiver bandwidth equals 13 dB; this fixes the power of the ASE that has been added to the signal along the transmission line. The dotted curves in Fig. 1 show the PDs of ONE and ZERO, found from the exact equation (15) for the aforementioned value of the output OSNR and in the case where there is no RIN. This baseline case is referred to in what follows as "ASE only." The solid curves, labeled "ASE+RIN" show the respective PDs obtained by numerical solution of (17) with all parameters being as stated previously and, in addition, for $\mathcal{D} = 2$. The latter value corresponds to the case where the amount of the noise power added to the signal by the transferred RIN equals the noise power contributed by the ASE. Indeed, from (11) and (12), $(1/2)\mathcal{D}$ equals the ratio of the amounts of the transferred RIN and the ASE (recall that the channel power $\bar{P}_{s,\text{in}}$ has been normalized to unity). Now, in order to answer the question posed in Section I regarding the relative effect of the RIN and ASE on the BER, we also plotted the PDs obtained from (15) in which we set the output OSNR to

$$\frac{1}{8\tilde{D}_{\mathcal{N}z}} = \frac{1}{8D_{\mathcal{N}z}} \cdot \frac{1}{1+\mathcal{D}/2}. \quad (21)$$

The corresponding dashed curves in Fig. 1 are labeled "equivalent amount of ASE," because they are obtained from the RIN-free equation (15) in which the amount of the ASE is set equal to the sum of the amount of the ASE actually present at the output *plus* the amount of the transferred RIN.

[2]It also depends on the filter bandwidth of the receiver, but this is outside the scope of the idealized detector model we consider here.

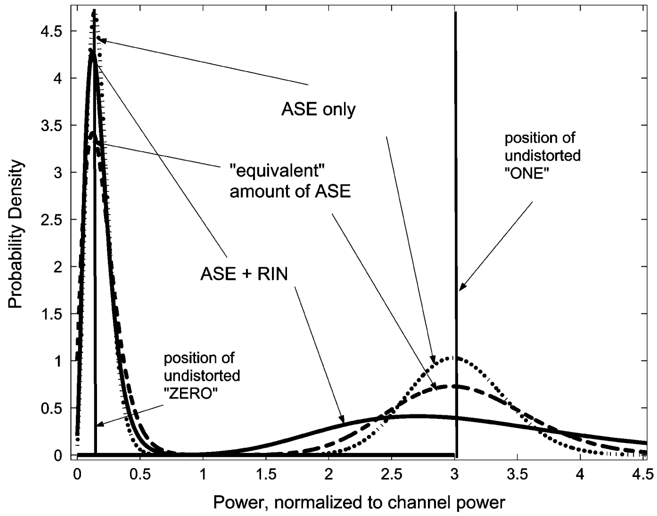

Fig. 1. Probability densities for logical ONE and ZERO. For a detailed description of parameters and notations, see text preceding (21). The ratio "RIN/ASE" of the powers of the transferred RIN and the ASE is 100%.

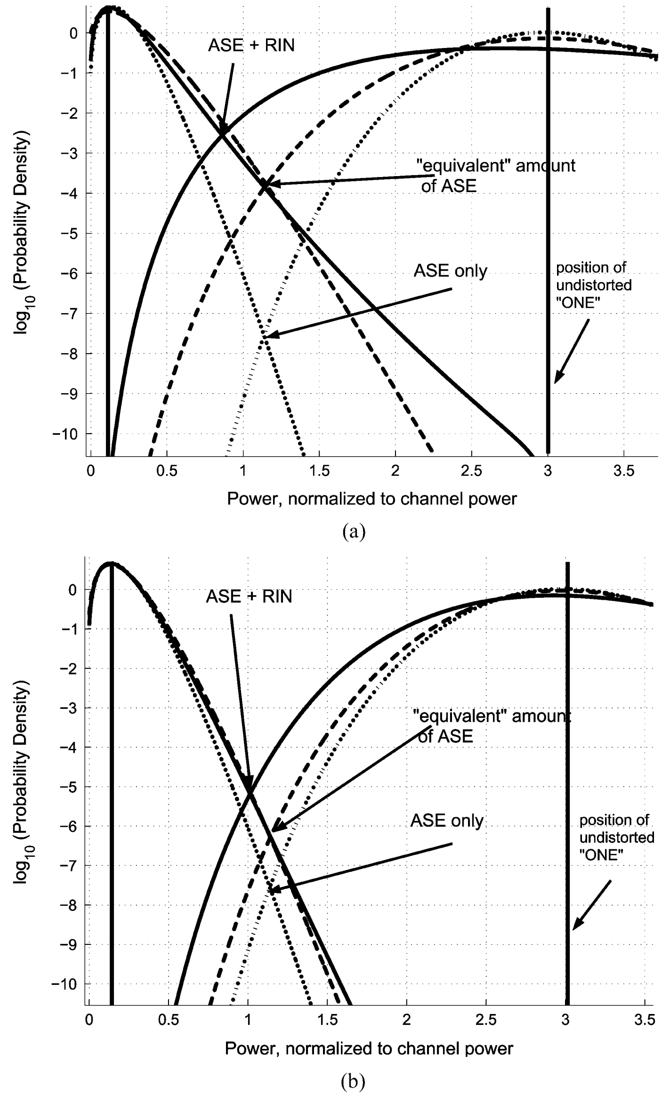

Fig. 2. (a) Same as in Fig. 1 but plotted in the logarithmic scale. (b) Same as in (a), but the ratio "RIN/ASE" is 20%.

The BER in each of the considered cases is estimated from the formula

$$\mathrm{BER} = \min_{w_0^{\mathrm{ZERO}} \le w_{\mathrm{th}} \le w_0^{\mathrm{ONE}}} \left[\int_0^{w_{\mathrm{th}}} \rho^{\mathrm{ONE}}(w)\, dw + \int_{w_{\mathrm{th}}}^{\infty} \rho^{\mathrm{ZERO}}(w)\, dw \right] \quad (22)$$

where the minimum is obtained by numerically scanning through possible values of the threshold w_{th}. In order to show the "tails" of ρ^{ONE} and ρ^{ZERO}, which determine the BER, we replot Fig. 1 in the logarithmic scale in Fig. 2(a). The BER values corresponding to the three different cases shown there are $\mathrm{BER}_{\text{"ASE only"}} = 1.1 \cdot 10^{-9}$, $\mathrm{BER}_{\text{"ASE+RIN"}} = 2.6 \cdot 10^{-4}$, $\mathrm{BER}_{\text{"equivalent amount of ASE"}} = 1.2 \cdot 10^{-5}$. Thus, the BER in the case where both the ASE and the transferred RIN degrade the signal is about 20 times worse than the BER in the case where the *same* total amount of noise is contributed by the ASE only.

Note also that the above value for the $\mathrm{BER}_{\text{"ASE only"}}$ indicates that the idealized detection model used here produces results that are close to those obtained with a more realistic model. Namely, for a pseudorandom bit sequence of 33% Gaussian pulses with an extinction ratio of 13 dB, detected by a receiver with a 30-GHz optical filter and a 7.5-GHz electrical fourth-order Bessel filter, the OSNR required to obtain a BER of 10^{-9} is about 13.8 dB, according to the Virtual Photonics (VPI) simulator. This is quite close to the 13-dB output OSNR, which we use to obtain the $\mathrm{BER}_{\text{"ASE only"}}$ above.

The situation where the amounts of noise contributed by the transferred RIN and by the ASE are equal, which is reported in Figs. 1 and 2(a), exaggerates the RIN contribution that is likely to occur in a typical transmission system. Fig. 2(b) shows the results for a more realistic case where the ratio of the powers of noise contributed by the RIN and the ASE is 20%. For this and other ratios "RIN/ASE," the corresponding values of the BER for the cases "ASE+RIN" and "equivalent amount of ASE," defined previously, are shown in Fig. 3 by thick lines. (Note that

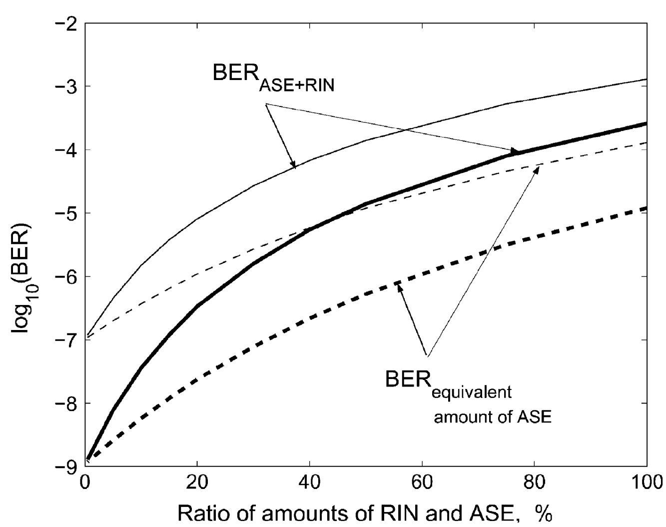

Fig. 3. Solid lines: BER values obtained with the theory of this paper. Dashed lines: BER values obtained by treating the transferred RIN as the same amount of ASE and using (15). Thick lines: Baseline $\mathrm{BER}_{\text{ASE only}} = 10^{-9}$. Thin lines: Baseline $\mathrm{BER}_{\text{ASE only}} = 10^{-7}$.

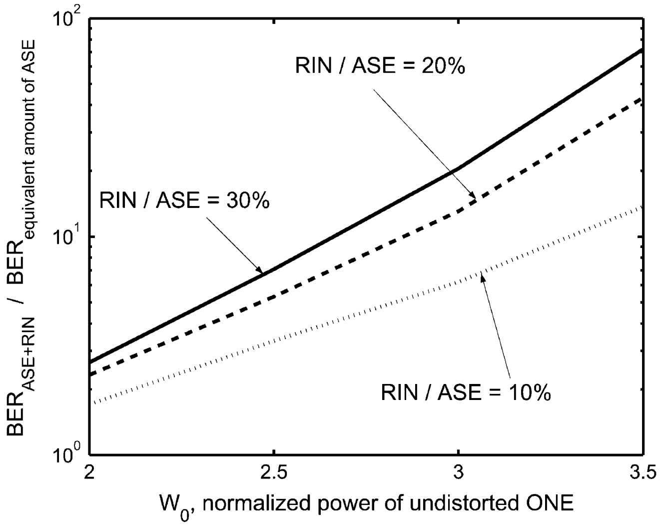

Fig. 4. Ratio of values of the BER computed with the theory of this manuscript and by treating the transferred RIN as the same amount of ASE, plotted as a function of the undistorted ONE's power. Solid, dashed, and dotted lines correspond to values of "RIN/ASE" equal 30%, 20%, and 10%. The baseline $\mathrm{BER}_{\mathrm{ASE\ only}} = 10^{-9}$, and the power of an undistorted ZERO is 13 dB below the power of an undistorted ONE.

$\mathrm{BER}_{\text{“ASE only”}} = 1.1 \cdot 10^{-9}$ and does not change with the ratio "RIN/ASE".) The main conclusion following from Fig. 3 is that for practically important values of the ratio "RIN/ASE," i.e., those between 10% and 30%, the BER computed using the assumption that the transferred RIN can be identified with an equal amount of the ASE, is at least an order of magnitude better than the BER calculated using the more accurate theory presented in this paper.

The BER also depends on the amplitude w_0 of the undistorted ONE. Fig. 4 shows the BER values plotted as a function of w_0 for three realistic values of the ratio "RIN/ASE." As was mentioned previously, $w_0 = 3$ corresponds to pulses occupying 33% of the bit slot. Furthermore, w_0 decreases as the duty ratio increases so that $w_0 = 2.5$ corresponds to 50% pulses, which is another widely used modulation format. In Fig. 4, we also presented results for $w_0 = 2$ and $w_0 = 3.5$ to account for possible situations where output pulses may become broader or shorter than the input ones due to an interplay between the fiber nonlinearity and residual dispersion, due to filtering, or due to suboptimal dispersion postcompensation at the receiver. The main conclusion from Fig. 4 is that the difference between the values of $\mathrm{BER}_{\text{“ASE+RIN”}}$ and $\mathrm{BER}_{\text{“equivalent amount of ASE”}}$ increases with the increase of w_0. In addition, this difference is more pronounced for higher ratios "RIN/ASE," as can be expected.

Modern transmission systems use forward-error correction of the detected signal, which allows one to bring a BER value down by several orders of magnitude. Therefore, it is appropriate to compare the effects of the transferred RIN and an equal amount of the ASE in the case where the baseline $\mathrm{BER}_{\text{“ASE only”}}$ is, say, 10^{-7} (the corresponding required output OSNR is 11.78 dB) instead of 10^{-9}. The results are shown in Fig. 3 with thin lines. The main conclusion from that figure is that the difference between the BER computed using the approach of this paper and that computed by identifying the transferred RIN with an equal amount of the ASE decreases as the baseline BER increases. This is to be expected, since the aforementioned difference occurs due to the *"tails"* of the PD being significantly different in those two cases. As the baseline BER increases, the corresponding PDs of the ONEs and ZEROs become broader and, hence, the probability of error at the decision threshold becomes less sensitive to the shape of the "tails" of the PDs and more determined solely by the variances of the corresponding quantities. Those variances, in turn, are determined only by the total amount of noise and thus, by definition, are the same in the cases "ASE+RIN" and "equivalent amount of ASE."

Finally, we note that the FPE (13) predicts that the BER of the detected signal will depend not only on the amount of transferred RIN but also on *where in the line* the RIN was added to the signal. Mathematically, this occurs because the ASE- and RIN-related operators $[(1/r)\partial/\partial r(r\partial/\partial r)$ and $(1/r)\partial/\partial r(r\partial/\partial r r^2)$, respectively] on the RHS of (13) do not commute. Physically, the reason is that the amount of noise transferred to the signal by the RIN at a given z depends on how much noise has already been accumulated up to the distance z [cf. (6)]. However, to ensure an acceptably small BER, the total noise power at the receiver bandwidth should be at least an order of magnitude lower than the output signal power and, hence, the signal field s on the RHS of (6) can be approximated by its input value s_0. This shows that the aforementioned dependence on where along the line the RIN was transferred to the signal should be weak. Table I confirms this expectation. The data in Table I are obtained for the ratio "RIN/ASE" equal 30%, and the rest of the parameters are the same as for Fig. 3.

IV. Conclusion

In this study, we have investigated the effect that the RIN transferred to the signal from a Raman co-pump has on the BER. Our main conclusion can be briefly stated as follows: corrupting the signal with RIN worsens the BER more than doing so with an equal amount of the ASE.

The quantitative measure of *how much worse* the BER gets depends on a number of factors. First, the difference between the BERs $\mathrm{BER}_{\mathrm{ASE+RIN}}$ and $\mathrm{BER}_{\mathrm{equivalent\ amount\ of\ ASE}}$ in the above two cases depends on the total power of the ASE in the receiver bandwidth and also on its ratio to the amount of the transferred RIN. When the total ASE power increases, thus leading to an increase of the baseline BER attained without the RIN contribution, the difference between the $\mathrm{BER}_{\mathrm{ASE+RIN}}$ and $\mathrm{BER}_{\mathrm{equivalent\ amount\ of\ ASE}}$ decreases; compare the thick and thin lines in Fig. 3. When the ratio of the accumulated RIN and ASE powers increases from zero to about 20%, the aforementioned difference also increases; compare the solid and dashed lines in Fig. 3. It is remarkable, however, that that difference remains almost the same as the ratio of the RIN and ASE powers changes between 20% and 100%.

Second, the difference between the $\mathrm{BER}_{\mathrm{ASE+RIN}}$ and $\mathrm{BER}_{\mathrm{equivalent\ amount\ of\ ASE}}$ increases as the pulses get narrower (see Fig. 4). For typical values of the pulse duty ratio of 33% of the bit slot (cooresponding to $w_0 \approx 3$ in Fig. 4), the RIN-to-ASE power ratio of 20%, and the baseline $\mathrm{BER}_{\mathrm{ASE\ only}} = 10^{-7}$, the aforementioned difference of the

TABLE I
VALUES OF $\text{BER}_{\text{“ASE+RIN”}}$ WHEN THE SAME TOTAL AMOUNT OF RIN IS ACCUMULATED AT DIFFERENT LOCATIONS ALONG THE TRANSMISSION LINE

Baseline BER	RIN is added uniformly	RIN is added only in 1st half	RIN is added only in 2nd half
$\text{BER}_{\text{“ASE only”}} = 10^{-9}$	$1.6 \cdot 10^{-6}$	$1.6 \cdot 10^{-6}$	$1.7 \cdot 10^{-6}$
$\text{BER}_{\text{“ASE only”}} = 10^{-7}$	$2.7 \cdot 10^{-5}$	$2.6 \cdot 10^{-5}$	$2.8 \cdot 10^{-5}$

BERs is about one order of magnitude. This difference is almost insensitive to where in the transmission line the RIN gets transferred to the signal (see Table I).

Finally, we would like to note that an analysis similar to the above can also be performed to calculate the effect of the RIN on the BER for the differential-phase-shift keying signal modulation scheme. The mathematical details of such an analysis are more involved than those for the ON–OFF keying scheme considered here, and this problem is currently being studied by the present author.

APPENDIX A

Here, we demonstrate that the statistics of the quantity $\int_0^L P_R(z,t)\,dz$, appearing in (1), is nearly Gaussian when P_R is a counter-pump. The same conclusion will also hold for a co-pump if the walk-off τ_{wo} between it and the signal is much greater than the duration of the signal pulse τ_p.

Notice that the signal depends on t via the combination $\tau = t - (1/V_s)z$

$$\int_0^L P_R(z,t)\,dz = \int_0^L P_R\left(z, \tau - \left(\frac{1}{V_P} - \frac{1}{V_s}\right)z\right)dz \tag{A1}$$

where V_s, V_P are the signal's and pump's group velocities. Then we can write in the order-of-magnitude sense

$$\int_0^L P_R\left(z, \tau - \left(\frac{1}{V_P} - \frac{1}{V_s}\right)z\right)dz \sim \sum_{n=0}^{\tau_{\text{wo}}/\tau_p} P_R(z, \tau - n\tau_p)\left(\frac{L\tau_p}{\tau_{\text{wo}}}\right) \tag{A2}$$

where the walk-off $\tau_{\text{wo}} = L((1/V_P) - (1/V_S))$. For a counter-pump, $V_P \approx -V_S$, and $\tau_{\text{wo}} \sim L/V_S$. Thus, for $L \sim 20$ km, the walk-off between a counter-pump and the signal is about 100 μs. For $\tau_p < 100$ ps, the summation in (A2) is over $\sim 10^6$ terms, and then by the central limit theorem, the statistics of the RHS of that equation is very close to Gaussian.

APPENDIX B

The PD given by (15) can be derived in more than one way. For example, it can be obtained by taking the joint PD $\rho(\text{Re}[s] = r\cos\varphi, \text{Im}[s] = r\sin\varphi)$, which is Gaussian in both $\text{Re}[s]$ and $\text{Im}[s]$, and integrating it over φ. Here we describe another approach, which does not require the knowledge of $\rho(\text{Re}[s], \text{Im}[s])$.

Let us seek the solution of (13) with $\mathcal{D} = 0$ in the form $\rho(r,z) = \int_0^\infty e^{-\nu^2 D_{\mathcal{N}} z} R(\nu r)\,d\nu$. Substituting this expansion into (13) with $\mathcal{D} = 0$, one obtains

$$\frac{d^2R}{d\tilde{r}^2} + \frac{1}{\tilde{r}}\frac{dR}{d\tilde{r}} + R = 0, \quad \tilde{r} = \nu r \tag{B1}$$

whence $R = J_0(\nu r)\hat{\rho}(\nu)$, where $J_0(x)$ is the Bessel function of zeroth order and $\hat{\rho}(\nu)$ is an arbitrary function of ν. Combining the above with the initial condition (14) yields

$$\frac{1}{r}\delta(r - r_0) = \int_0^\infty J_0(\nu r)\hat{\rho}(\nu)\,d\nu. \tag{B2}$$

Multiplying (B2) by $rJ_0(\mu r)$ and using the orthogonality relation

$$\int_0^\infty J_0(\nu r)J_0(\mu r)r\,dr = \frac{1}{\nu}\delta(\nu - \mu)$$

one obtains $\hat{\rho}(\nu) = \nu J_0(\nu r_0)$. (Recall that in (14), as well as in (B2), $r_0 > 0$. Otherwise, i.e., if $r_0 = 0$, the above calculation is invalid, e.g., $\hat{\rho}(\nu) = \nu$, and the integral in (B2) does not exist even in the sense of distributions.) Finally, one substitutes the above expression for $\hat{\rho}$ into $\rho(r,z) = \int_0^\infty e^{-\nu^2 D_{\mathcal{N}} z}\hat{\rho}(\nu)J_0(\nu r)\,d\nu$ and uses the formula [16]

$$\int_0^\infty e^{-\nu^2 x}J_0(\nu r)J_0(\nu r_0)\nu\,d\nu = \frac{1}{2x}\exp\left(-\frac{r^2 + r_0^2}{4x}\right)I_0\left(\frac{rr_0}{2x}\right)$$

to arrive at (15).

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T. I. Lakoba, photograph and biography not available at the time of publication.